Level UP MATHS HOMEWORK BOOK

Author team: Greg and Lynn Byrd
Contents

Introduction

Unit 1 An opening sequence
☐ 1.1 Generating terms in a sequence
☐ 1.2 Linear sequences
☐ 1.3 Quadratic and fraction sequences
☐ 1.4 Sequences from patterns
☐ 1.5 Functions and mappings
☐ 1.6 World solar challenge

Unit 2 Keep your balance
☐ 2.1 Using letter symbols in algebra
☐ 2.2 Constructing and solving linear equations
☐ 2.3 Solving linear equations with \( x \) and brackets on both sides
☐ 2.4 Using trial and improvement to solve equations
☐ 2.5 Solving simultaneous equations graphically
☐ 2.6 Solving simultaneous equations algebraically

Unit 3 Share and share alike
☐ 3.1 Decimals and fractions in order
☐ 3.2 Adding and subtracting fractions
☐ 3.3 Multiplying and dividing fractions
☐ 3.4 Using percentages
☐ 3.5 Get things in proportion
☐ 3.6 Ratio and proportion
☐ 3.7 Brain power
☐ 3.8 More calculation strategies
☐ 3.9 Efficient calculation

Unit 4 Be constructive
☐ 4.1 Quadrilaterals, angles and proof
☐ 4.2 Interior and exterior angles in polygons
☐ 4.3 Angles in triangles and quadrilaterals
☐ 4.4 Pythagoras’ theorem
☐ 4.5 Using Pythagoras’ theorem to solve problems
☐ 4.6 Constructions
☐ 4.7 Construction problems
☐ 4.8 Circles and tangents
☐ 4.9 Perilous paper round

Unit 5 Stat’s entertainment
☐ 5.1 Planning an investigation
☐ 5.2 Organising data
☐ 5.3 Calculating and using statistics
☐ 5.4 Pie charts
☐ 5.5 Line graphs
☐ 5.6 Frequency diagrams and frequency polygons

Unit 6 Extreme measures
☐ 6.1 Converting between measures
☐ 6.2 Compound measures and bounds
☐ 6.3 Circles and perimeter
☐ 6.4 Circles, sectors and area
☐ 6.5 Area and volume
☐ 6.6 Volume and surface area

Unit 7 Power up
☐ 7.1 Labyrinth
☐ 7.2 To round or not to round?
☐ 7.3 Using a calculator
☐ 7.4 More rounding
☐ 7.5 Roots and standard form on a calculator
☐ 7.6 Written methods
☐ 7.7 Using a calculator efficiently
☐ 7.8 Problem solving
Unit 8 Graphic detail

- 8.1 Prime factors, HCF and LCM
- 8.2 Using factors and multiples
- 8.3 Multiplying and dividing with indices
- 8.4 Index laws with negative and fractional powers
- 8.5 Reviewing straight-line graphs
- 8.6 Investigating the properties of straight-line graphs
- 8.7 Graphs of quadratic and cubic functions
- 8.8 Functions and graphs in real life
- 8.9 Investigating patterns

Unit 9 Strictly come chancing

- 9.1 Fair play
- 9.2 It's exclusive!
- 9.3 Tree diagrams
- 9.4 Relative frequency

Unit 10 Shape shifter

- 10.1 Testing for congruence
- 10.2 Transforming shapes
- 10.3 UFO hunt
- 10.4 Similarity
- 10.5 Introducing trigonometry
- 10.6 Using trigonometry 1

Unit 11 Magic formula

- 11.1 Simplifying algebraic expressions
- 11.2 Working with double brackets
- 11.3 Factorising quadratics
- 11.4 Using and writing formulae
- 11.5 Working with formulae
- 11.6 Inequalities

Unit 12 No problem?

- 12.1 Data problem solving
- 12.2 Number problem solving
- 12.3 Algebra problem solving

- 12.4 Geometrical reasoning: lines, angles and shapes
- 12.5 Percentage and proportion problems
- 12.6 Functions and graphs

Unit 13 Data day statistics

- 13.1 Questionnaires and samples
- 13.2 Averages from grouped data
- 13.3 Classroom challenge
- 13.4 Scatter graphs and correlation
- 13.5 Misleading graphs and charts
- 13.6 Comparing distributions

Unit 14 Trig or treat?

- 14.1 Solving geometrical problems
- 14.2 More geometrical problems
- 14.3 3-D shapes
- 14.4 Prisms and cylinders
- 14.5 Using trigonometry 2
- 14.6 Solving problems in trigonometry

Unit 15 A likely story

- 15.1 This will probably be familiar!
- 15.2 Growing trees
- 15.3 Experiment!
- 15.4 What's the problem?
- 15.5 Back to the future

Unit 16 Dramatic mathematics

- Unit 16 Homework investigation 1: What a pane!
- Unit 16 Homework investigation 2: What's 4 T?
- Unit 16 Homework investigation 3: What has 52 bones and 250 000 pores...?
- Unit 16 Homework investigation 4: Have you met your match?
- Unit 16 Homework investigation 5: Simple symbol sums!
- Unit 16 Homework investigation 6: Cash flow
Welcome to Level Up Maths!

Level Up Maths is an inspirational new course for today's classroom. With stunning textbooks and amazing software, Key Stage 3 Maths has simply never looked this good!

The Homework Book has 16 units, with one homework page for each lesson in the Level Up 6–8 Textbook. The homework questions cover the same topics as the textbook pages, at the same levels.

Every homework starts with a question to practise your number skills.

Your teacher may tell you to tick the questions to try.

The sub-levelled questions practise the topics covered in the lesson.

This shows where to look for help on the LiveText CD.

13.5 Misleading graphs and charts

1. The sum of two consecutive multiples of 6 is 222. Use an algebraic method to find the value of these integers.

2. The table shows data on the favourite sports of the Year 8 girls in one school. Which type of graph or chart would make it easiest for the PE teacher to see what fraction of the girls liked the different sports?

3. This bar chart shows information about the numbers of merits given to the boys and girls of BT in the last four weeks. John says, 'The girls got twice as many merits in week 4 as they did in week 2.' Explain why John is wrong.

4. This bar chart shows information about the number of merits given to the boys and girls of BT in the last four days.
   a. What is wrong with this bar chart?
   b. How could you improve it?

5. Miss Needham gave her English class a simple crossword to do. She recorded how long it took each pupil to finish. The results are shown in the bar chart. Use the bar chart to say whether each of these statements is true or false, or whether there is not enough information to tell. Explain your answers.
   a. The median time was between 5 and 6 minutes.
   b. The fastest person took 3 minutes.
   c. No one took more than 8 minutes.
   d. The girls got more correct answers than the boys.
   e. The boys finished, on average, faster than the girls.

This shows you the games to play on the LiveText CD. (Not for every homework.)
The LiveText CD in the back of this book has:

- The whole textbook on screen.

**Explanations, to help you understand the Big Ideas.**

Glossary to explain maths words. Play audio to hear translations in Bengali, Gujarati, Punjabi, Turkish and Urdu.

- Games to practise your maths skills.

**Walk the Plank**
1.1 Generating terms in a sequence

1. The answers to these calculations can be found on the cards below.
   
   $80 - (24 - 3^2)$  
   $(2 + 4)^2 + 10 \div 5$  
   $7^2 - 2 \times 7$  
   $60 - 4^2 + 2$  
   $(5 \times 2)^2 - 36 \div 2$  
   $6 + 3 \times 5$

   Work out the answers to the calculations.
   Now write down the letters from the cards with the answers on, starting with the smallest answer.
   What word have you written.

   52 M  45 K  64 E  92 T  21 B  82 S
   66 P  38 D  37 T  51 R  65 A  35 I

2. Use these term-to-term rules to find the first five terms of each sequence.
   a. Start at -5, add 2, then multiply by 2.
   b. Start at 50, divide by 2, then subtract 3.

3. Match each term-to-term definition to the correct position-to-term definition.
   
   Term-to-term  Position-to-term
   a. Start at 0, add 5 each time.  Multiply the term number by 2, then add 3.
   b. Start at 5, add 2 each time.  Multiply the term number by 5, then subtract 5.
   c. Start at 2, add 5 each time.  Multiply the term number by 5, then subtract 3.

4. Write down the first four terms of each sequence.
   a. $T(n) = 4n - 1$
   b. $T(n) = 1 - 4n$

5. Use the pattern of first differences to generate the next term in each sequence.
   a. 7, 8, 10, 13, ...
   b. -4, 0, 6, 14, ...

6. Generate the first four terms of each sequence.
   a. $T(n) = n^2 + 10$
   b. $T(n) = \frac{1}{2}(n^2 - 1)$

7. The $n$th term of a sequence is given by

   $T(n) = (n + 1)(n + 2) + n$

   Write down the first five terms.
1.2 Linear sequences

1. In a café some of the prices on the blackboard have been rubbed out. The price of a coffee and one cake is £2.85. The price of a coffee and two cakes is £4.20. Work out the price of a coffee and the price of a cake.

2. Here is a pattern made from sticks.

3. Here is a pattern made from sticks.

4. For each sequence work out the $n$th term using the pattern of differences.
   a. 0, 10, 20, 30, 40, ...
   b. 4, 14, 24, 34, 44, ...
   c. 70, 60, 50, 40, 30, ...

5. Here is a pattern made from sticks.

   The value of each term in the sequence is the number of sticks used.
   a. Work out the $n$th term in the sequence.
   b. Explain your $n$th term rule by reference to the structure of the pattern.

6. Design your own pattern sequence. Design one you have not used before. Work out the $n$th term and explain it by reference to the physical pattern.
1.3 Quadratic and fraction sequences

1. Copy these numbers. They are a coded message!

```
5 7 3 4 1 2 5 6 2 3 28 10 5 7 3 7 4 5 16 28 2 3 21 1 2 1 3
```

**Key:**
- \( \frac{2}{3} \times 15 = H \)
- \( \frac{7}{15} - \frac{2}{15} = S \)
- \( \frac{1}{5} + \frac{3}{5} = A \)
- \( \frac{12}{15} = U \)
- \( \frac{7}{8} \times 32 = T \)
- \( \frac{7}{8} - \frac{2}{8} = N \)
- \( \frac{1}{12} + \frac{9}{12} = W \)
- \( \frac{3}{5} \times 35 = O \)
- \( \frac{1}{21} + \frac{13}{21} = F \)

Each number in the code is the answer to one of the calculations in the key. Replace each number with the matching letter to crack the code. The first one has been done for you.

\( \frac{2}{3} \times 15 = 10 \)
So 10 represents H.

What is the secret message?

2. Find five consecutive terms for each of these expressions.
   - a) \( n^2 \)
   - b) \( 10n^2 \)
   - c) \( n(n + 2) \)
   - d) \( n^2 + n + 1 \)

3. Use your answers to Q2 to work out the difference pattern for each of these expressions.
   - a) \( n^2 \)
   - b) \( 10n^2 \)
   - c) \( n(n + 2) \)
   - d) \( n^2 + n + 1 \)

4. Each of these quadratic sequences has its \( n \)th term in the form \( T(n) = an^2 \).
   Use the pattern of differences to find the \( n \)th term.
   - a) 2, 8, 18, 32, 50, ...
   - b) 4, 16, 36, 64, 100, ...
   - c) \( \frac{1}{2}, 4, \frac{1}{2}, 8, 12 \frac{1}{2}, \ldots \)

5. Each of these quadratic sequences has its \( n \)th term in the form \( T(n) = an^2 + b \).
   Use the pattern of differences to find the \( n \)th term.
   - a) 11, 14, 19, 26, 35, ...
   - b) \(-1, 5, 15, 29, 47, \ldots \)

6. Each of these quadratic sequences has its \( n \)th term in the form \( T(n) = an^2 + bn + c \).
   Use the pattern of differences to find the \( n \)th term.
   - a) 6, 13, 24, 39, 58, ...
   - b) \(-4, -3, 0, 5, 12, \ldots \)

7. Work out the rule for the \( n \)th term for each of these fraction sequences.
   - a) \( \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \ldots \)
   - b) \( 0, \frac{1}{6}, \frac{2}{9}, \frac{3}{12}, \ldots \)
   - c) \( \frac{1}{2}, \frac{3}{5}, \frac{6}{10}, \frac{10}{17}, \ldots \)
   - d) \( \frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \frac{5}{50}, \ldots \)

1.4 Sequences from patterns

1. Copy and complete these.
   a. $4.6 \times 10 = \square$
   b. $530 \div \square = 0.53$
   c. $0.077 \times \square = 7.7$

2. Here are the first five terms of a sequence of rectangular numbers.

   By looking at ways of dividing the rectangular pattern of dots, deduce that $T(n) = n^2 + n$

3. Here are the first five terms of a sequence of rectangular numbers.

   By looking at ways of dividing the rectangular pattern of dots, deduce that $T(n) = (n - 1)^2 + 3n - 1$

4. Here is a sequence of patterns made from dots.

   a. Write each one of these terms as an arithmetic series.
   b. Write down the $n$th term, $T(n)$, as an arithmetic series.

5. The drawing shows another way of dividing one of the rectangular patterns in Q2.

   It shows that
   $1 + 3 + 5 + 7 + 9 = 4^2 + 4$

   a. Repeat for two of the other rectangular patterns in Q2.
   b. Deduce the general formula for this type of rectangular pattern, using the sum of the first $n$ odd numbers.

6. A conservatory manufacturer always makes conservatories such that the number of floor tiles needed along the length of the floor is 5 more than along the width.

   The diagrams show the first two sizes of conservatory.

   a. Predict the number of tiles in the next size.
   b. Draw the diagram for the next size to test your prediction.
   c. Work out the rule for the number of tiles in the $n$th size.
   d. Justify your rule by looking at the pattern of floor tiles.
1.5 Functions and mappings

1. Work out the mean of £8, £12, £5 and £3.
2. Write each of these function machines as an equation.
   a. $x \rightarrow \times 2 \rightarrow +5 \rightarrow y$
   b. $x \rightarrow +5 \rightarrow \times 2 \rightarrow y$
   c. $x \rightarrow \times 2 \rightarrow +5 \rightarrow y$
3. Draw a mapping diagram for each function for $x = 0$ to $x = 5$.
   a. $x \rightarrow 2x - 3$
   b. $x \rightarrow 3 - 2x$
   c. $x \rightarrow \frac{x}{2} + \frac{1}{2}$
4. Using suitable positive and negative values of $x$, plot the graph of each function.
   a. $x \rightarrow 3x$
   b. $x \rightarrow 2x - 3$
   c. $x \rightarrow \frac{x}{2} - 2$
5. Find the inverse of each function.
   a. $x \rightarrow 3x$
   b. $x \rightarrow 2x - 3$
   c. $x \rightarrow \frac{x}{2} - 2$
6. Match each red graph to the blue graph of its inverse function.

- A: $y = x$
- B: $y = \frac{1}{3}x$
- C: $y = 3x$
- D: $y = -3x$
- E: $y = 2x$
- F: $y = \frac{1}{3}x$
- G: $y = x$
- H: $y = -\frac{1}{3}x$
7. These are graphs of quadratic functions.
   a. $y = \frac{1}{2}x^2 + 2$
   b. $y = -x^2 - 2$

   i. For each graph write down the equation of the line of symmetry.
   ii. For each graph write down the coordinates of the turning point, saying whether it is a maximum or a minimum.
1. Write down the first four terms of the sequence whose \( n \)th term is:

- \( T(n) = 3n + 6 \)
- \( T(n) = 1 - 2n \)
- \( T(n) = n^2 + 4 \)
- \( T(n) = 100 - n^2 \)

2. Your solar car has an on-board computer. The computer keeps a record of the charge the batteries are getting from the solar cells. Match each statement with the most appropriate graph.

- a Sunny day with a brief electrical fault that stops charging to the batteries until fixed.
- b Sunny day with car passing through the shadow of a cloud.
- c Sunny day with car passing through the shadow of a bridge.
- d Charge from sunrise on a sunny day.
- e Charge until sunset on a sunny day.
- f Hazy day with brief period of clear sunshine.

3. Your car is to be tested in a hilly area. The track diagram shows the distance, in metres, to each checkpoint, and the arrival times in seconds.

<table>
<thead>
<tr>
<th>Start</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Finish</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>600 m</td>
<td>15 s</td>
<td>1500 m</td>
<td>60 s</td>
<td>1800 m</td>
<td>120 s</td>
</tr>
</tbody>
</table>

- a Plot the distance–time graph for this test. Use the \( x \)-axis to represent time, from 0 to 170 seconds. Use the \( y \)-axis to represent distance, from 0 to 3400 metres.
- b Which section of the track do you think was the steepest downhill section?
- c Work out the speed of the car on the steepest uphill section of the track.
- d Work out the average speed of the car for the entire test. Give your answer, in metres per second (m/s), correct to one decimal place.
2.1 Using letter symbols in algebra

1. Write true or false for each of these.
   a. $-8 + (-3) = 11$
   b. $7 + (-16) = -9$
   c. $-6 + 10 - (-3) = 1$

2. The green cards below are a mixture of expressions, equations and identities. Put each card into the correct group. The first one is done for you.

   **Expressions**
   - $4x - 12$
   - $2x + 4 = x + x + 4$
   - $4x = 7(2x + 1)$

   **Equations**
   - $6 - 3x = 2x + 1$
   - $16x - 5n + 4t$
   - $4p - 3s = 4x + 3x + 3x$

   **Identities**
   - $8x + 12 = 28$
   - $8p - 3q + 7x + 3w$
   - $3(y - 2) + 5 = 3y - 1$

3. Juan sent $t$ text messages last week. Carlos sent 5 fewer than Juan. Rico sent $q$ more than Carlos.
   a. Write an expression for the total number of text messages sent by Juan, Carlos and Rico. Simplify your expression.
   b. Damita sent 5 more than double the number Juan sent. Write an expression for the number of text messages sent by Damita.
   c. Find an expression for the difference between the total number of text messages sent by Juan, Carlos and Rico, and the number sent by Damita. Simplify your expression.

4. a. Write an expression for the combined area of both orange shapes using brackets.
   b. Expand and simplify your answer to part a.

5. Expand these.
   a. $4x(3x + 6y)$
   b. $2x^2(3y - 4x)$
   c. $xy^2(4xy + 3x^2)$

6. This is part of Yvonne’s homework. Mark it and correct any mistakes she has made.

   **Expand the brackets and simplify.**
   a. $3(x + y) + 2(x - y) = 3x + y + 2x - y = 5x$
   b. $7(x - y) - 4(x + y) = 7x - 7y - 4x - 4y = 3x - 3y$
   c. $8x(x + y) - 4(x^2 - 3xy) = 8x^2 + 8xy - 4x^2 - 12xy = 4x^2 - 4xy$

7. Expand and simplify these.
   a. $p(2p + 3d - 1) + 4p(5d - 3 + 2w)$
   b. $b^2(3b + 2c^2 + 4f) - 2b(b^2 - 5be^2 + bf)$
2.2 Constructing and solving linear equations

1. This is Kyle's homework on estimating square roots. Write G for 'good estimate' or B for 'bad estimate' for each of his answers. If you think he has made a bad estimate, write down a good one.

   Estimate these square roots.
   a. \( \sqrt{10} \) estimate = 3.1
   b. \( \sqrt{52} \) estimate = 7.2
   c. \( \sqrt{150} \) estimate = 11.8
   d. \( \sqrt{23} \) estimate = 4.8

2. This triangle and this rectangle have the same perimeter. The side lengths are measured in centimetres.
   a. Form an equation in terms of \( n \).
   b. Solve the equation to find the value of \( n \).
   c. Work out the perimeter of the rectangle and triangle.

3. Triangle \( ABC \) is a right-angled triangle. The perimeter of the triangle is 48 cm.
   a. Form an equation in terms of \( x \).
   b. Solve the equation to find the value of \( y \), and write down the lengths of the sides of the triangle.
   c. Form an equation in terms of \( x \).
   d. Solve the equation to find the value of \( x \), and write down the sizes of the angles in the triangle.

4. Lynn has two dogs, Deefa and Barney. Barney is \( x \) years old. Deefa is three years older than Barney. Lynn is eight years older than twice the total age of Deefa and Barney. Lynn is 42 years old.
   a. Form an equation in terms of \( x \).
   b. Solve your equation and write down the ages of Deefa and Barney.

5. The perimeters of a regular pentagon and an equilateral triangle are equal. The pentagon has side lengths, in centimetres, of \( 3x - 6 \). The triangle has side lengths, in centimetres, of \( 2x + 11 \).
   a. Form an equation, and solve it to find \( x \).
   b. Work out the side lengths of the pentagon and the triangle.
   c. Work out the perimeter of the pentagon and the triangle.

6. I think of a number, multiply it by 3 then add 16. The answer is the same if I subtract 4 from the number, multiply the result by 6 and then subtract 2. Form an equation, and solve it to find the number I thought of.
2.3 Solving linear equations with \(x\) and brackets on both sides

1. Tao is going to have a test on converting between metric and imperial units. When he starts to revise he spills orange juice on the conversion table in his book. Copy the table and fill in the numbers that Tao cannot see.

   **Imperial to metric conversion**
   
   - 1 mile \(\approx\) \(x\) km
   - 1 foot (ft) \(\approx\) \(x\) cm
   - 1 pound (lb) \(\approx\) \(x\) kg
   - 1 ounce (oz) \(\approx\) \(x\) g
   - 1 pint \(\approx\) \(x\) litres
   - 1 gallon \(\approx\) \(x\) litres

2. Match the equations on these cards into equivalent pairs.
   
   A: \(6 - 3x = 2x + 1\)
   B: \(2x = 8\)
   C: \(12 - 14x = 13x\)
   D: \(5x = 6\)
   E: \(8x - 7 = 1 + 3x\)
   F: \(3x + 4 = x + 12\)
   G: \(2 + 6x = 30x - 10\)
   H: \(5x = 5\)

3. Copy and complete these to make equivalent equations.
   
   a. \(7x + 2 = 3x + 9\) \(x + 2 = 9\)
   b. \(8x - 3 = 5 - x\) \(x - 3 = 5\)
   c. \(5x + 9 = 2x + 24\) \(x = \) \(\) \(\)
   d. \(4(x - 1) = 6 - 2x\) \(x = \) \(\)

4. Solve these equations.
   
   a. \(6x + 4 = 3x + 13\)
   b. \(9x - 5 = 7x + 5\)
   c. \(x + 14 = 22 - 3x\)
   d. \(8 - 4x = 4x - 8\)

5. The diagram shows a parallelogram. All lengths are in centimetres.
   
   a. Write down two equations using the facts given.
   b. Solve your equations to find the values of \(x\) and \(y\).
   c. Work out the perimeter of the parallelogram.

6. Which of these three equations gives the ‘odd solution out’?
   
   A: \(3(x + 5) = x + 25\)
   B: \(3(2x - 5) = 2(x + 6) - 5\)
   C: \(4(x - 2) = 3(9 - x)\)

7. Solve these equations.
   
   a. \(2(4x - 1) = 7x - 5(x + 4)\)
   b. \(6 - 3(4 - 2x) = 7(x - 3)\)
   c. \(5 + 2(2x + 3) = 23 - 4(3x - 5)\)
2.4 Using trial and improvement to solve equations

1. Write 0.36 as a fraction in its simplest form.

2. The length of a rectangle is 5 cm more than the width. The area of the rectangle is 92 cm². Use trial and improvement to work out the width of the rectangle to one decimal place. The first trial has been done for you.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Area</th>
<th>Too big or too small?</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x + 5</td>
<td>x(x + 5)</td>
<td>4 x 9 = 36</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>4 x 9</td>
<td>36</td>
</tr>
</tbody>
</table>

3. The height of a triangle is 2 cm less than the length of the base. The area of the triangle is 79 cm². Use trial and improvement to work out the height of the triangle to one decimal place.

4. Use trial and improvement to find the solution to the equation \( x^3 = 780 \) to one decimal place.

5. A solution to the equation \( x^3 + 3x^2 - 750 = 0 \) lies between 8 and 9. Use trial and improvement to find a solution to the equation to two decimal places.

6. Gareth is designing the carton for an orange juice drink. The carton must be a cuboid with a square base. The height of the cuboid must be 10 cm more than the side length of the base. The carton must hold 500 ml when full. Gareth tries to work out the side length of the base of the carton to one decimal place. This is what he has written so far.

Let the side length of the base = \( x \). So the height of the cuboid = \( x + 10 \).
Volume = \( x^2(x + 10) = x^3 + 10 \). Volume must equal 500 ml = 500 cm³.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x^3 + 10 )</th>
<th>Too big or too small?</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>226</td>
<td>too small</td>
</tr>
</tbody>
</table>

a. What is the one mistake that Gareth has made?
b. Re-write Gareth’s solution, and work out the side length of the base of the carton to one decimal place.

7. Gareth also designs the carton for a banana smoothie. This carton is also a cuboid and must hold 750 ml when full. The base length must be 3 cm more than the width, \( y \), and the height must be 5 cm more than the width. Form an equation and solve it, using trial and improvement, to find \( y \) to one decimal place.
2.5 Solving simultaneous equations graphically

1. Lars took part in a sponsored walk. He raised £400 for charity. He gave £80 to the RSPB, £220 to the RSPCA and the rest to the PDSA. What fraction of the total money did Lars give to each charity? Give your answers in their lowest terms.

2. Complete this table of values for the line \( y = 8 - x \).
   \[
   \begin{array}{|c|c|c|c|}
   \hline
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
   \hline
   \end{array}
   \]
   a Complete a table of values for the line \( y = 3x \) for values of \( x \) from 0 to 4.
   b Use your tables of values to draw the lines \( y = 8 - x \) and \( y = 3x \).
   c Write down the coordinates of the point where your lines cross.

3. Complete this table of values for the line \( y + 2x = 8 \).
   \[
   \begin{array}{|c|c|c|c|}
   \hline
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
   \hline
   \end{array}
   \]
   a Complete a table of values for the line \( y - 3x = 3 \) for values of \( x \) from 0 to 4.
   b Use your tables of values to draw the lines \( y + 2x = 8 \) and \( y - 3x = 3 \).
   c Write down the coordinates of the point where your lines cross.

4. Use the graph to solve each pair of simultaneous equations.
   Write ‘no solution’ if the lines do not cross and ‘infinite number of solutions’ if the lines are equivalent.
   a \( x + y = 5 \) and \( y = 4x \)
   b \( x + y = 5 \) and \( x + y = 3 \)
   c \( x + y = 3 \) and \( y = x + 1 \)
   d \( y = x + 1 \) and \( y - x = 1 \)

5. An electricity company has two ways of charging customers for their electricity.
   **Tariff A:**
   standing charge of £14 plus £14 for every 100 units of electricity used
   **Tariff B:**
   no standing charge but £18 for every 100 units of electricity used
   a Construct a table of values for Tariff A and Tariff B, showing the cost of electricity for 0, 100, 200, 300, 400 and 500 units.
   b Use your table of values to draw two straight-line graphs.
   c Write the equations of both lines.
   d Use your graph to find the number of units of electricity for which both tariffs would charge the same.

6. Solve each pair of simultaneous equations using a graphical method.
   a \( y = x + 2 \) and \( y = 4 - x \)
   b \( y = 2x - 1 \) and \( y + 3x = 9 \)

---

Play any game on the LiveText CD.
2.6 Solving simultaneous equations algebraically

1. The probability of Sue picking a toffee at random from a bag of sweets is 0.48. What is the probability that Sue picks a sweet at random from the bag that is not a toffee?

2. Solve each pair of simultaneous equations.
   a. $x + 3y = 22$ and $x + y = 10$
   b. $x - 2y = 1$ and $x + 2y = 13$
   c. $4x + 3y = 32$ and $-x - 3y = -26$
   d. $2x + 7y = 24$ and $5x + 7y = 39$

3. a. Copy and complete these to make equivalent equations.
      $2x + y = 17$
      $4x + 2y = \square$
   b. Use your answer to part a to help solve this pair of simultaneous equations.
      $2x + y = 17$ and $3x + 2y = 28$

4. Solve each pair of simultaneous equations.
   a. $3x + y = 10$ and $2x + 5y = 24$
   b. $3x - y = 21$ and $x + 5y = 23$
   c. $-2x + 3y = 7$ and $-5x + y = -28$
   d. $4x + 3y = 5$ and $2x + 7y = 19$

5. Bryn has spilt cola on his homework. Copy Bryn’s homework and fill in the missing parts.

6. Delyth has made a mistake in her homework. Explain what mistake she has made, and complete her homework correctly.

7. a. Write down a pair of simultaneous equations from this parallelogram.
   b. Solve your equations to find $x$ and $y$.
   c. Work out the angles in the triangle.
3.1 Decimals and fractions in order

1. Which of the numbers in the cloud below are square numbers?

2. By converting the fractions to decimals, write these fractions in ascending order.

3. Use equivalent fractions to write these fractions in ascending order.

4. a. Write these negative decimal numbers in descending order.

   b. How many of the numbers in part a are greater than -6.5?

5. Write true or false for each of these.

   a. 3.566 > 3.656
   b. 6.77 < 6.599
   c. 0.03389 > 0.0337
   d. 28.989 > 28.99

6. Put the correct inequality sign between each pair of temperatures.

   The first one is done for you.

   a. -12.55°C < 7.95°C
   b. -3.57°C < -3.75°C
   c. -8.82°C > -8.8°C
   d. -0.084°C < -0.009°C
   e. -1.07°C > -1.7°C

7. Lynn measured the height of her puppy when it was 3 months old and when it was 6 months old.

   The heights were $\frac{7}{25}$ m and $\frac{1}{2}$ m.

   Which height do you think is the height of the puppy at 3 months old, and which is the height of the puppy at 6 months old?

   Give reasons for your answers.
3.2 Adding and subtracting fractions

1. Work out these.
   a) $7 - 11$
   b) $-8 + 2$
   c) $-9 - 3$
   d) $-11 + 20$

2. Work out these additions.
   a) $1\frac{1}{3} + 4\frac{5}{6}$
   b) $2\frac{5}{6} + 3\frac{7}{15}$
   c) $12\frac{3}{8} + 6\frac{2}{3}$
   d) $9\frac{6}{7} + \frac{2}{3}$
   e) Add together two and three-fifths and two and a quarter.

3. Work out these subtractions.
   a) $3\frac{2}{3} - 1\frac{1}{7}$
   b) $5\frac{1}{2} - 2\frac{3}{10}$
   c) $12\frac{7}{8} - \frac{5}{2}$
   d) $22\frac{2}{3} - 17\frac{1}{4}$
   e) Take one and five-sixths from three and a third.

4. Copy the table below.
   Match the question on the left, to the workings in the middle, to the answer on the right. The first one is done for you.

<table>
<thead>
<tr>
<th>Question</th>
<th>Working</th>
<th>Working</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $2\frac{1}{3} - \frac{3}{4}$</td>
<td>$\frac{10}{4} - \frac{9}{8}$</td>
<td>$\frac{53}{12} - \frac{22}{12}$</td>
<td>$3\frac{5}{8}$</td>
</tr>
<tr>
<td>b) $3\frac{2}{6} - 1\frac{2}{3}$</td>
<td>$\frac{23}{8} - \frac{7}{4}$</td>
<td>$\frac{41}{18} - \frac{14}{18}$</td>
<td>$1\frac{1}{18}$</td>
</tr>
<tr>
<td>c) $4\frac{5}{12} - 1\frac{3}{10}$</td>
<td>$\frac{7}{3} - \frac{3}{4}$</td>
<td>$\frac{43}{10} - \frac{14}{10}$</td>
<td>$2\frac{3}{8}$</td>
</tr>
<tr>
<td>d) $3\frac{4}{5} - 1\frac{3}{16}$</td>
<td>$\frac{41}{18} - \frac{7}{6}$</td>
<td>$\frac{30}{9} - \frac{9}{9}$</td>
<td>$1\frac{7}{12}$</td>
</tr>
<tr>
<td>e) $2\frac{5}{18} - \frac{7}{9}$</td>
<td>$\frac{28}{9} - \frac{2}{3}$</td>
<td>$\frac{38}{12} - \frac{9}{12}$</td>
<td>$1\frac{7}{12}$</td>
</tr>
<tr>
<td>f) $\frac{5\frac{3}{8} - \frac{12}{4}}{8}$</td>
<td>$\frac{53}{12} - \frac{12}{12}$</td>
<td>$\frac{56}{18} - \frac{27}{18}$</td>
<td>$1\frac{1}{2}$</td>
</tr>
</tbody>
</table>

5. Work out these.
   a) $\frac{5}{12} + \frac{35}{18} - \frac{2}{3}$
   b) $7\frac{1}{3} - 2\frac{2}{5} + 3\frac{9}{20}$

6. Simon is training for a marathon. On Monday he runs $6\frac{2}{3}$ miles, on Wednesday he runs $8\frac{1}{2}$ miles, on Friday he runs $5\frac{3}{6}$ miles and on Sunday he runs $12\frac{2}{9}$ miles. What is the total distance that he runs in this week?

7. Abbie spends $\frac{3}{8}$ of her birthday money on guitar lessons, $\frac{1}{6}$ on books and $\frac{2}{15}$ on chocolates.
   a) What fraction of her birthday money does she have left?
   b) She gives half of the money she has left to her brother. What fraction of her birthday money does Abbie give to her brother?
3.3 Multiplying and dividing fractions

1. Write down all the factor pairs of 32.

2. Here are some results of the survey of 3600 pet owners.
   - a) \(\frac{1}{2}\) would like more pets.
   - b) \(\frac{1}{4}\) had guinea pigs as pets.
   - c) \(\frac{3}{4}\) preferred their pets to most people.
   - d) \(\frac{5}{6}\) love spoiling their pets.
   - e) \(\frac{11}{12}\) had their pet from a rescue centre.
   How many pet owners are included in each result?

3. Work out these. Simplify your calculation whenever possible.
   - a) \(\frac{5}{3}\) of 75 g
   - b) \(\frac{5}{6}\) of £248
   - c) \(\frac{1}{2}\) of 436 m
   - d) \(\frac{11}{30}\) of 690 kg

4. a) Match each red card with the correct yellow card.

   - 12 ÷ \(\frac{1}{2}\)
   - 8 ÷ \(\frac{1}{4}\)
   - 25
   - 3 ÷ \(\frac{1}{7}\)
   - 24
   - 32
   - 6 ÷ \(\frac{1}{3}\)
   - 16
   - 18

   b) Which yellow card is left over?
   c) Write a calculation for this red card that would give the answer on the yellow card that is left over.

5. Which is the correct answer for each of these, A, B or C?
   - a) The reciprocal of \(\frac{1}{2}\) is
     - A \(\frac{1}{2}\)
     - B \(\frac{1}{4}\)
     - C \(\frac{2}{3}\)
   - b) The reciprocal of \(\frac{2}{3}\) is
     - A \(\frac{3}{2}\)
     - B \(\frac{1}{6}\)
     - C \(\frac{2}{3}\)
   - c) The reciprocal of \(\frac{2}{3}\) is
     - A \(\frac{3}{2}\)
     - B \(\frac{2}{3}\)
     - C \(\frac{3}{2}\)
   - d) The reciprocal of 6 is
     - A \(\frac{1}{6}\)
     - B \(\frac{1}{6}\)
     - C \(\frac{1}{2}\)

6. Copy and complete these divisions.
   Write your answer in its simplest form and as a mixed number if appropriate.
   - a) \(\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}\)
   - b) \(\frac{2}{3} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2} = \frac{1}{1} = 1\)
   - c) \(1\frac{1}{2} \div \frac{1}{3} = \frac{3}{2} \times \frac{3}{1} = \frac{9}{2} = 4\frac{1}{2}\)
   - d) \(\frac{2}{3} \div \frac{3}{10} = \frac{2}{3} \times \frac{10}{3} = \frac{20}{9} = 2\frac{2}{9}\)

7. In his will Mr Jones left a sum of money to be shared among four charities. The RSPCA had \(\frac{1}{3}\), the RSPB had \(\frac{1}{6}\), the PDSA had \(\frac{1}{4}\) of what was left, and the Cats Protection League had the remainder.
   What fraction of the sum of money did the Cats Protection League have?
3.4 Using percentages

1. Write true or false for each of these.
   a) \( \frac{2}{7} + \frac{3}{7} = \frac{7}{14} \)
   b) \( \frac{4}{5} + \frac{2}{5} - \frac{1}{5} = 1 \)
   c) \( \frac{4}{9} + \frac{4}{9} - \frac{5}{9} = \frac{1}{3} \)

2. Jed and Jake are waiters at different restaurants.
   Jed is paid £6.10 per hour and is about to be given a 10% pay rise.
   Jake is paid £5.80 per hour and is about to be given a 15% pay rise.
   a) How much will Jed and Jake earn per hour after their pay rise?
   b) After their pay rise, who will earn the most and by how much?
   c) One week, they both work for 40 hours. What is the difference in their earnings?

3. a) Carlos sells his flat for £103 200. He sells it for 120% of the original cost.
    So £103 200 represents 120%.
    i) What is 1%?
    ii) Work out the original cost of the flat.
   b) Mair sells her house for £136 000. She sells it for 15% less than the original cost.
    i) What percentage of the original cost of the house is £136 000?
    ii) What is 1%?
    iii) Work out the original cost of the house.

4. Match each green percentage increase/decrease card with the correct pink multiplier card.
   - 20% increase
   - 0.8
   - 12% decrease
   - 1.2
   - 2% increase
   - 0.98
   - 2% decrease
   - 1.2
   - 20% decrease
   - 0.98

5. a) After Christmas Tanya weighs 75.6 kg. This is 8% more than she weighed before Christmas. Calculate how much Tanya weighed before Christmas.
   b) Dylan goes on a diet. He loses 12% of his body weight. He now weighs 79.2 kg. Calculate Dylan’s body weight before he started the diet.

6. Megan has £2000 in the bank.
   She invests it for 3 years at a rate of 4% compound interest.
   a) How much does Megan have in the bank at the end of the 3 years?
   b) How much interest has Megan received by the end of the 3 years?

7. Which is the correct calculation needed to work out the total value of each of these investments, A, B or C?
   a) £1500 invested for 2 years at 3% compound interest.
      A 1500 \times 0.03^2
      B 1500 \times 1.02^3
      C 1500 \times 1.03^2
   b) £3600 invested for 3 years at 4.5% compound interest.
      A 3600 \times 1.05^3
      B 3600 \times 1.045^3
      C 3600 \times 0.045^3
   c) £8000 invested for 6 years at 6% compound interest.
      A 8000 \times 1.06^6
      B 8000 \times 0.06^6
      C 8000 \times 1.06^6
3.5 Get things in proportion

1. Work out these.
   a. $4 \times -3$
   b. $12 \div -4$
   c. $-25 \times -2$
   d. $-10 \div 2$

2. Two supermarkets have a special offer on jars of instant coffee.
   - **Supermarket A**
     - £1 off all jars of coffee
   - **Supermarket B**
     - 20% off all jars of coffee

Which supermarket's special offer represents a proportional change?

3. Which of these adverts for diet pills represent a proportional weight loss?
   - **A** ‘Weigh-lite’ pills
     - Lose $\frac{1}{10}$ of your body weight in only 1 month!
   - **B** ‘Diet-doctor’ pills
     - Lose 2 kg of your body weight in only 2 weeks!
   - **C** ‘Easy-diet’ pills
     - Lose 5% of your body weight in only 1 week!

4. Which of these are not always in direct proportion? Give a reason for your answer.
   - A the age of a person and their weight
   - B the height in metres and the height in feet
   - C the price in euros and the price in pounds
   - D the age of a car and its value

5. In which of these tables is $y$ proportional to $x^2$?
   - **A**
     - $\begin{array}{|c|c|c|c|c|}
        \hline
        x & 1 & 2 & 3 & 4 \\
        y & 10 & 13 & 18 & 25 \\
        \hline
     \end{array}$
   - **B**
     - $\begin{array}{|c|c|c|c|c|}
        \hline
        x & 1 & 2 & 3 & 4 \\
        y & 2 & 8 & 18 & 32 \\
        \hline
     \end{array}$
   - **C**
     - $\begin{array}{|c|c|c|c|c|}
        \hline
        x & 1 & 2 & 3 & 4 \\
        y & 12 & 9 & 6 & 3 \\
        \hline
     \end{array}$

6. In each coordinate pair $(a, b)$, $b$ is proportional to the inverse of $a$, that is $b \propto \frac{1}{a}$.
   - a. Use these coordinate pairs to work out a formula that expresses $b$ in terms of $a$.
     - $(2, \frac{1}{2})$, $(3, 1)$, $(4, \frac{1}{2})$, $(5, \frac{3}{5})$
   - b. Which of these coordinate pairs do not fit the formula from part a?
     - $(7, \frac{2}{3})$, $(9, 3)$, $(12, \frac{1}{2})$, $(25, \frac{3}{25})$, $(27, \frac{1}{3})$, $(30, 10)$

7. Write true or false for each of these.
   - a. If $x$ is the side length of a square, and $y$ is the area of six of the squares, then $y \propto x^2$.
   - b. If $x$ is the side length of an equilateral triangle, and $y$ is the perimeter of the equilateral triangle, then $y \propto x$.
   - c. If $x$ is the number of cm, and $y$ is the number of inches, then $y \propto \frac{1}{x}$.
   - d. If $x$ is the number of km, and $y$ is the number of kg, then $y$ is not proportional to $x$.
   - e. If $x$ is the number of people, and $y$ is their total weight then $y \propto x$. 

2.5 Get things in proportion
3.6 Ratio and proportion

1. Work out these.
   a) 1/2 of £32  
   b) 1/3 of 60 kg  
   c) 2/3 of 15 cm  
   d) 5/7 of $21

2. Rhian, Ffion and Ellen share prize money of £6400 in the ratio 2 : 3 : 5. How much does each of them receive?

3. Write each of these ratios in its simplest form.
   a) 4 cm : 5 m  
   b) 25p : £6  
   c) 15 hours : 2 days  
   d) 8 months : 3 years

4. Ricky is mixing paint.
   He makes a light blue paint by mixing 2 tins of white paint with 3 tins of blue paint.
   a) Write a ratio to show the number of tins of white paint to the number of tins of blue paint.
   He makes a light green paint by mixing 3 tins of white paint with 5 tins of green paint.
   b) Write a ratio to show the number of tins of white paint to the number of tins of green paint.
   c) Express each ratio in the form 1 : m.
      Which paint has the higher proportion of colour in it?
   d) Rewrite each ratio in the form m : 1. What does the m stand for in this ratio?

5. Here are two circles.
   The radius of the smaller circle is r and the radius of the larger circle is R.
   The ratio of r to R is 1 : 3.
   a) What is the ratio of the area of the smaller circle to the area of the larger circle?
   b) The area of the smaller circle is 12 cm². What is the area of the larger circle?

6. Here are two mathematically similar triangles.
   The area of triangle A is 12 cm² and the area of triangle B is 192 cm².
   a) Write the ratio of the area of triangle A to the area of triangle B in the form 1 : m.
   b) What is the ratio of the length of the sides of triangle A to the length of the sides of triangle B?
   c) The height of triangle B is 12 cm. What is the height of triangle A?

7. The ratio of the lengths of the sides of two similar cuboids is 1 : 2.
   a) What is the ratio of the areas of the bases?
   b) What is the ratio of the volumes?
   c) The volume of the larger cuboid is 104 cm³. What is the volume of the smaller cuboid?
3.7 Brain power

Answer these questions without using a calculator.

1. Find the highest common factor (HCF) of 36 and 63.
2. Match each blue card with the correct yellow card.
   - \((3 + 4 \times 2)^2\) 98
   - 60 - (27 - 13)
   - \((9 - 5)^2 + 24\)
   - \(8^2 \times 2 - 30\)
   - \(150 + 7^2 - 12 \div 4\)
   - \(121\)

3. Work out the answer to each of these.
   - a) \(3^3 - 7 \times 2\)
   - b) \(5^3 - 5^2 - 5\)
   - c) \((6^2 - 2 \times 17)^3\)
   - d) \((2 + \sqrt{64})^2\)

4. Which is the correct answer for each of these, A, B or C?
   - a) \(\frac{4 + 4^2}{5 - 3}\) A 1  B 10  C 32
   - b) \(\frac{(2 + 3)^2}{5 + 20}\) A 1  B 10  C 25
   - c) \(\frac{6^2 \times 2}{3 \times -3}\) A -12  B -8  C 8
   - d) \(\frac{6 + 3^2 \times 2}{1 + 3^2}\) A 6  B 6.6  C 69

5. This is part of Owen's homework. He has both the answers wrong.
   - a) \(6 + 5 \times 6 = 11 \times 6 = 66 = 33\)
   - b) \(4^2 + 16 = 64 + 16 = 80 = -20\)

Explain the mistakes that Owen has made, and work out the correct answers for him.

6. Copy the table below. Match the question on the left with the working in the middle and the answer on the right. The first one is done for you.

<table>
<thead>
<tr>
<th>Question</th>
<th>Working</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (\sqrt{12})</td>
<td>(\sqrt{25} \times \sqrt{3})</td>
<td>(3 \times \sqrt{2})</td>
</tr>
<tr>
<td>b) (\sqrt{18})</td>
<td>(\sqrt{121} \times \sqrt{2})</td>
<td>(7 \times \sqrt{2})</td>
</tr>
<tr>
<td>c) (\sqrt{75})</td>
<td>(\sqrt{4} \times \sqrt{3})</td>
<td>(11 \times \sqrt{2})</td>
</tr>
<tr>
<td>d) (\sqrt{98})</td>
<td>(\sqrt{9} \times \sqrt{2})</td>
<td>(2 \times \sqrt{3})</td>
</tr>
<tr>
<td>e) (\sqrt{242})</td>
<td>(\sqrt{49} \times \sqrt{2})</td>
<td>(5 \times \sqrt{3})</td>
</tr>
</tbody>
</table>

7. Simplify each of these expressions. a) \(\sqrt{2} \times \sqrt{72}\)  b) \(\sqrt{5} \times \sqrt{45}\)
3.8 More calculation strategies

1. Find the lowest common multiple (LCM) of 12 and 30.

2. Copy and complete these.
   a. \( \frac{1}{100} = 0.01, \) so \( \frac{7}{100} = 0.01 \times 7 = \) ___
   b. \( \frac{1}{20} = 0.05, \) so \( \frac{3}{20} = 0.05 \times 3 = \) ___
   c. \( \frac{1}{8} = 0.125, \) so \( \frac{5}{8} = \) ___ \times ___ = ___
   d. \( \frac{1}{5} = \) ___ , so \( \frac{3}{5} = \) ___ \times ___ = ___
   e. \( \frac{1}{80} = 0.0125, \) so \( \frac{3}{80} = \) ___ \times ___ = ___
   f. \( \frac{1}{500} = \) ___ , so \( \frac{7}{500} = \) ___ \times ___ = ___

3. Fill in the gaps in these statements, using numbers from the cloud.
   a. 10% = 0.1, so 30% = ___
   b. 1% = ___ , so 9% = ___
   c. ___ % = 0.15, so ___% = 0.45
   d. 1.5% = ___ , so 71.5% = ___
   e. Which number from the cloud haven’t you used?
   f. Write this number as a percentage.

4. This is part of Tom’s homework. Decide which questions he has got right and which he has got wrong.
   For those that are wrong, write out the correct solution.
   a. \( \frac{3}{4} \) of 18 = \( 0.8 \times 18 = 8 \times 18 \div 10 = 14.4 \)
   b. \( \frac{2}{5} \) of 25 = \( 0.7 \times 25 = 7 \times 25 \div 10 = 15 \)
   c. \( \frac{3}{8} \) of 12 = \( 0.3 \times 12 = 3 \times 12 \div 10 = 3.6 \)

5. Copy and complete these.
   a. 0.25 \times 36 = \( \frac{1}{4} \) of 36 = 36 \div ___ = ___
   b. 0.3 \times 24 = \( \frac{3}{10} \) of 24 = 24 \div ___ \times ___ = ___
   c. 0.04 \times 720 = \( \frac{4}{100} \) of 720 = 720 \div ___ \times ___ = ___
   d. 0.125 \times 400 = \( \frac{125}{1000} \times 400 = 400 \div ___ \times ___ = ___

6. Dave goes on holiday to America when the exchange rate is \$1.98 = \£1.
   Caroline goes on holiday to America when the exchange rate is \$1.72 = \£1.
   They both exchange \£200 into dollars.
   How many more dollars does Dave receive than Caroline?

7. The ‘Lucky Dip’ box shown is in the shape of a cube.
   The volume of the box is 0.125 m³.
   a. What is the length of one side of the box?
   b. What is the total area that is painted?
   The box is painted red on four of its sides.
3.9 Efficient calculation

1. Eleanor raises £360 for charity in a sponsored swim. She gives £90 to the local hospital, £120 to an animal rescue centre and the rest to cancer research.
   a. How much does Eleanor give to cancer research?
   b. What fraction of the £360 does Eleanor give to the local hospital?
   c. What fraction of the £360 does Eleanor give to the animal rescue centre?
   d. What fraction of the £360 does Eleanor give to cancer research?
   Give your answers to parts b, c and d in their lowest terms.

2. Work out these.
   a. 62 × 0.1
   b. 62 × 0.01
   c. 3.8 × 0.1
   d. 38 × 0.01
   e. 120 × 0.1
   f. 0.5 × 0.01
   g. 369 × 0.1
   h. 12,000 × 0.01

3. Copy and complete. Insert the correct sign, <, > or =, between each pair.
   a. 35 ÷ 0.1 ______ 350 ÷ 0.1
   b. 600 ÷ 0.01 ______ 6.1 + 0.1
   c. 780 ÷ 0.1 ______ 0.78 ÷ 0.01
   d. 82 ÷ 0.01 ______ 0.082 ÷ 0.01

4. Which is the correct answer for each of these, A, B or C?
   a. 0.6 × 0.4 ______ A. 2.4
      b. 0.02 × 0.3 ______ B. 0.24
      c. 0.15 ÷ 0.02 ______ C. 0.004
      d. 1.2 ÷ 0.9 ______ A. 1.08
         ______ B. 0.108

5. Match each pink card with the correct yellow card.

6. Estimate the answers to these calculations, by rounding each number to 1 s.f.
   a. 7.21² + 83.35 × 18.711
   b. 71.5 + 203.6

7. This is part of Sue's homework. She has got both the answers wrong.
   Explain the mistakes that Sue has made, and work out the correct estimates for her.
1. The first term of a sequence is 4. The term-to-term rule is subtract 2, then multiply by 3. Work out the first five terms of the sequence.

2. For each of the diagrams below, write down the letters of the pairs of angles that are corresponding angles.

3. Calculate the size of the lettered angles. Give reasons for your answers.

4. Copy and complete this proof to show that the exterior angle of a triangle is equal to the sum of the two interior opposite angles.

   \[ w + y + \ldots = 180^\circ \text{ because angles in a triangle sum to } \ldots \]

   \[ x + \ldots = 180^\circ \text{ because they lie on } a \ldots \text{ line.} \]

   So \( w + y + z = x + \ldots \)

   So \( w + z = \ldots \)

5. Decide whether each statement is a definition, a convention or a derived property.
   
   a. A triangle has three sides and three interior angles.
   b. Parallel lines are marked with arrows pointing in the same direction.
   c. A right angle is marked using a small square.
   d. Angles in a triangle sum to 180°.

6. Is this a practical demonstration or a proof?

   Kristof cuts out a parallelogram. He tears the corners off and arranges them around a point to show that the angles sum to 360°.

7. Is this a practical demonstration or a proof?

   There are 180° in a triangle. Two triangles join together to make a quadrilateral, so the angles in a quadrilateral sum to 360°.
4.2 Interior and exterior angles in polygons

1. Use the fact that $16 \times 34.5 = 552$ to work out these.
   a. $1.6 \times 3.45$
   b. $160 \times 345$
   c. $32 \times 34.5$

2. Calculate the sizes of the lettered angles in these shapes. State any angle facts that you use.

   a. \[ \angle a = 147°, \angle b = 76° \]
   b. \[ \angle a = 149°, \angle b = 125° \]

3. Explain how to find the size of the exterior angle of a regular octagon.

4. Calculate the size of
   a. the interior angle, and
   b. the exterior angle of a regular decagon.

5. The exterior angle of a regular polygon is $12°$.
   a. How many sides does the polygon have?
   b. Calculate the size of one of the interior angles.

6. A gardener designs the patio shown below. The patio consists of two squares with a regular octagon between them.

   Calculate the sum of all the interior angles of the patio.

7. The shape of a children's playground is shown below. It is the shape of a regular pentagon with two of the sections missing.

   Calculate the sum of the interior angles of the playground.
1. Sally and three friends are going ten-pin bowling. Each game will cost them £11 in total. They each have £15 and they are going to share the cost fairly.
   a. What is the greatest number of games they can play?
   b. If they play this number of games, how much money will they each have left?

2. Find the sizes of the lettered angles in these triangles.
   a. 
   b. 
   c. 

3. Look at this kite. \( \angle ABD = 74^\circ, \angle BDA = 37^\circ \)
   Work out the size of
   a. \( \angle DAC \)
   b. \( \angle BAC \)
   c. \( \angle ADC \)
   d. \( \angle ABC \)
   e. \( \angle BAD \)

4. In this diagram, lines \( AB \) and \( CD \) are parallel. \( \angle AEF = 42^\circ, \angle EGD = 117^\circ \)
   Calculate
   a. \( \angle EFG \)
   b. \( \angle EGF \)
   c. \( \angle FEG \)

5. In this shape, \( \angle ABC = 22^\circ \)
   Work out the size of
   a. \( \angle ACB \)
   b. \( \angle BAC \)

6. In this shape, \( \angle XW = 112^\circ, \angle WZY = 36^\circ \)
   Work out the size of
   a. \( \angle VXY \)
   b. \( \angle YWZ \)

7. This diagram shows a rhombus inside a rectangle. Calculate the value of \( x \) in this diagram. Explain your reasoning.
4.4 Pythagoras' theorem

1. This season the Little Haven bowls team won 15 matches, drew 9 and lost 6. Write the ratio of winning matches to drawn matches to lost matches in its simplest form.

2. Write true or false for each of these.
   a. The hypotenuse is the shortest side of a right-angled triangle.
   b. The hypotenuse is the side opposite the right angle.
   c. In the pink triangle, \( m \) is the hypotenuse.
   d. In the yellow triangle, \( f \) is not the hypotenuse.

3. a. Copy these numbers. They are a coded message.

<table>
<thead>
<tr>
<th>8.2</th>
<th>7.1</th>
<th>7.6</th>
<th>7.9</th>
<th>9.2</th>
<th>9.5</th>
<th>8.5</th>
<th>9.3</th>
<th>9.2</th>
<th>8.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9</td>
<td>8.1</td>
<td>8.6</td>
<td>9.2</td>
<td>8.1</td>
<td>7.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Each number in the code is the lettered length (to 1 d.p.) in one of the diagrams below. Replace each number with the matching letter to crack the code. The first one has been done for you.

   What is the secret message?

4. The coordinates of \( A \) are \((1, 2)\).
   The coordinates of \( B \) are \((6, 4)\).
   a. Write down the horizontal distance, \( a \).
   b. Write down the vertical distance, \( b \).
   c. Use Pythagoras' theorem to find the straight-line distance between \( A \) and \( B \).

5. Calculate the shortest distance between the points \((7, 11)\) and \((13, 18)\).
4.5 Using Pythagoras' theorem to solve problems

1. Without using a calculator, work out these.
   \[ \frac{5^2 + 11}{4^2 - 7} \quad \frac{6 \times 7}{2} - \sqrt{31} \quad (12 - 5)^2 - (20 + 3^2) \]

2. Write true or false for each of these.
   a. The numbers 5, 12 and 13 are a Pythagorean triple.
   b. The numbers 9, 12 and 15 are a Pythagorean triple.
   c. The numbers 15, 20 and 30 are a Pythagorean triple.
   d. The numbers 25, 60 and 65 are a Pythagorean triple.

3. A ladder, 6 m long, leans against a wall. The foot of the ladder is 1.5 m from the wall. How far up the wall does the ladder reach?

4. A flagpole, 8 m high, is supported by a metal wire of length 9 m. The metal wire goes in a straight line from the top of the flagpole, and is fixed to the ground. How far from the bottom of the flagpole is the wire fixed?

5. Greg lives in a cottage. This diagram shows the cross-section of the loft of his cottage. Greg puts loft insulation across the floor of his loft. How wide is the floor of his loft?

6. Sofia decides to keep chickens. She fences off a section of her garden for a chicken run. This is a sketch of the chicken run. What is the total length of fencing she needs to go all the way around the chicken run?

7. Jin decides to paint his garden shed. This is a sketch of the end wall of the shed. Calculate the area of the end wall.
In the stars are some fractions and some decimals. Match each decimal with its equivalent fraction.

Draw a straight line of length 10 cm. Using only a ruler and compasses, draw the perpendicular bisector of the line.

Use a protractor to draw an angle of 70°.
- Using only a ruler and compasses, draw the bisector of this angle.
- Use your protractor to check that the bisector you have drawn has divided the 70° angle into two 35° angles.

Using only a ruler and compasses, make an accurate drawing of this triangle.

Jane is having a conservatory put on the side of her house. This is a sketch of the conservatory roof.
- Make an accurate scale drawing of the roof using a scale of 1 cm : 0.5 m.
- Use a ruler to measure the height of the roof.
- Calculate the height of the roof using Pythagoras' theorem. Was your scale drawing accurate?

Rali is putting up a shelf in his bedroom. The shelf is supported by a bracket, as shown in the diagram.
- Make an accurate scale drawing of the bracket.
- Use a ruler to measure the length of the top of the bracket.
- By how many centimetres does the shelf overhang the bracket?

Issay draws a line AB 10 cm long. He then draws a circle of radius 6 cm from both ends of the line. His drawing looks like this. Issay says ‘The circles intersect in two places’. In how many places will the circles intersect if Issay draws the line AB 12 cm long? 14 cm long? Explain your answers.
4.7 Construction problems

1. Copy and complete this number pyramid. The number in each brick is the sum of the numbers in the two bricks below it. Use a written method, not a calculator.

2. a. Use a ruler and a protractor to draw triangle ABC accurately.
   b. Measure the length of side BC.

3. a. Use a ruler and compasses to draw quadrilateral WXYZ accurately.
   b. Measure the size of \( \angle XWZ \).

4. The diagram on the right is of two triangles, ABC and XYZ.
   \( AC = 10 \text{ cm}, BC = 6 \text{ cm}, CX = 3 \text{ cm} \)
   \( XY = 4 \text{ cm}, YA = 3 \text{ cm}, YZ = 5 \text{ cm} \)
   a. Use a ruler and compasses to draw this shape accurately.
   b. Work out the length of AZ.

5. a. Use a ruler and compasses to draw triangle EFG accurately.
   b. Measure the size of \( \angle EFG \).

6. Which of these triangles can you construct from the information given?
   a. \( AC = 10 \text{ cm}, BC = 7 \text{ cm}, \angle C = 42^\circ \)
   b. \( AC = 6 \text{ cm}, BC = 3 \text{ cm}, AB = 4 \text{ cm} \)
   c. \( AB = 8 \text{ cm}, BC = 5 \text{ cm}, \angle A = 55^\circ \)
   d. \( \angle A = 45^\circ, \angle B = 35^\circ, \angle C = 95^\circ \)
   e. \( BC = 9 \text{ cm}, \angle B = 85^\circ, \angle C = 40^\circ \)

7. Which of the triangles in Q6 are unique triangles? Explain your answers.
1. Two thirds of the people at a youth club are girls. Write down the ratio of boys to girls.

2. Match each part of the circle to the correct letter on the diagram.
   a. radius
   b. diameter
   c. chord
   d. circumference

3. This diagram shows a regular octagon ABCDEFGH inscribed in a circle.
   a. Work out the size of
      i. $\angle AOB$  
      ii. $\angle ABO$
      iii. $\angle DEF$  
      iv. $\angle HOE$
   b. Imagine a line drawn from A to C. What shape is ACDH?

4. a. Draw a circle of radius 5 cm.
    b. Mark any three points on the circumference of the circle and join them up to make a triangle.
    c. Construct the perpendicular bisector of each side of your triangle.
    d. Write down what you notice.

5. Copy and complete these statements. Use the words in the cloud.
   a. Lines that meet at right angles are ________.
   b. An ________ regular polygon is a polygon that fits exactly inside a ________.
   c. A ________ to a circle is a straight line that meets the circle at just ______ point. It is at right angles to the radius of the circle.
   d. The perpendicular from the centre of the circle to a chord ________ the chord.

6. In this diagram AB and CB are tangents to the circle and $\angle OBC = 18^\circ$.
   a. What is the size of $\angle OCB$?
      Explain your answer.
   b. Work out the size of $\angle AOC$.
      Show all your working.

7. In this diagram W is the mid-point of the line XY and $\angle XOW = 55^\circ$.
   a. What is the special name given to the line XY?
   b. Work out the size of
      i. $\angle WXO$  
      ii. $\angle WYO$  
      iii. $\angle XOY$
   c. What can you say about the lengths $OX$ and $OY$?
      Explain your reasoning.
   d. What can you say about the lengths $XW$ and $WY$?
      Explain your reasoning.
1. Work out these.
   a. \( \frac{2}{3} \times £78 \)
   b. \( \frac{2}{5} \times 48 \text{ kg} \)
   c. \( \frac{2}{5} \times 819 \text{ km} \)
   d. \( \frac{3}{7} \times 49 \text{ hours} \)

2. You’ve been so successful at delivering newspapers to Barkingside Terrace and Woolfbury Avenue that you have been given another street to deliver to: Thatwas Close. The last person to deliver there got it very wrong. All that was left of him was a half-eaten newspaper and a push-bike!

There are only six deliveries to be made, but as usual there are dogs in the gardens.

You need a safe route from the gate (G) to the letterbox (L).

Sliding rails are shown with a blue line.

This time there are safe route instructions for each garden, but they are all mixed up.

**Safe route instructions**

<table>
<thead>
<tr>
<th>A</th>
<th>(7, 0), (7, 3), (3, 6), (3, 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>(7, 0), (3, 3), (3, 10)</td>
</tr>
<tr>
<td>C</td>
<td>(7, 0), (7, 1), (1, 3), (1, 7), (3, 10)</td>
</tr>
<tr>
<td>D</td>
<td>(7, 0), (3, 10)</td>
</tr>
<tr>
<td>E</td>
<td>(7, 0), (2, 4), (3, 10)</td>
</tr>
<tr>
<td>F</td>
<td>Impossible without sausages. Stand at (7, 4) and throw them to (5, 7)</td>
</tr>
</tbody>
</table>

Match up the safe route instructions with the appropriate garden.

Each grid square is 1 metre wide.
5.1 Planning an investigation

1. Write true or false for each of these.
   - a) 4.33 > 4.303
   - b) 7.89 < 7.699
   - c) 0.02329 > 0.0233
   - d) 76.09 > 76.011

2. The Williams family live in Swansea. There are 12 secondary schools within 5 miles of where they live. Suggest possible sources of secondary data that they could use to find out about the different schools.

3. Look at this secondary data.
   UK average weekly earnings (before tax) from 2000 to 2006

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>England</td>
<td>£433</td>
<td>£460</td>
<td>£482</td>
<td>£498</td>
<td>£508</td>
<td>£526</td>
<td>£547</td>
</tr>
<tr>
<td>Wales</td>
<td>£373</td>
<td>£386</td>
<td>£405</td>
<td>£422</td>
<td>£438</td>
<td>£455</td>
<td>£470</td>
</tr>
<tr>
<td>Scotland</td>
<td>£389</td>
<td>£411</td>
<td>£435</td>
<td>£447</td>
<td>£456</td>
<td>£479</td>
<td>£504</td>
</tr>
<tr>
<td>Northern Ireland</td>
<td>£368</td>
<td>£382</td>
<td>£397</td>
<td>£412</td>
<td>£431</td>
<td>£451</td>
<td>£472</td>
</tr>
</tbody>
</table>

What could you investigate by using this data?

4. Kasia wants to know which sport people like to watch most on television. She asks the members of her athletics club. Give a reason why her sample could be biased.

5. Ian is doing a survey on healthy eating. She has written this question.
   
   **Do you agree that ‘take-away’ food is bad for you?**
   
   Agree [ ]  Don’t know [ ]

Write down two things that are wrong with Ian’s question.

6. Ian is doing a survey on public transport. He has written this question.
   
   **Do you think there are enough buses running during the rush-hour?**

   a) What is wrong with Ian’s question?
   b) Write a better question to find out people’s views on the number of buses running during the rush-hour.

7. The table shows the numbers of people of different ages living in a village in Wales. The village council want to ask 20% of the population to complete a survey about the facilities in the village. To avoid bias, how many people from each age group should be asked?

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of people</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 20</td>
<td>240</td>
</tr>
<tr>
<td>20 to 39</td>
<td>190</td>
</tr>
<tr>
<td>40 to 59</td>
<td>320</td>
</tr>
<tr>
<td>60 and over</td>
<td>370</td>
</tr>
<tr>
<td>Total</td>
<td>1120</td>
</tr>
</tbody>
</table>
5.2 Organising data

1. Work out these additions.
   
   a. $3\frac{1}{2} + 1\frac{1}{5}$
   
   b. $4\frac{3}{8} + 2\frac{3}{14}$

2. Diallo has recorded the numbers of runs scored by his cricket team in 40 matches. This is what he has written.

   64  124  101  92  137  96  107  135
   141  66  178  128  81  150  118  59
   82  166  158  98  115  120  132  111
   139  144  79  180  108  72  155  179
   88  106  122  110  145  133  89  114

   Construct a frequency table for Diallo’s data.

3. Rani runs a coffee shop. She sells four types of coffee: latte (L), mocha (M), americano (A) and cappuccino (C). She sells three types of cake: fruit (F), Bakewell (B) and vanilla (V). Rani records the type of coffee that customers buy as well as the type of cake they buy. This is what she writes.

   MF  AF  CV  LF  CB  CF  MB  MV  CB  CV
   LB  MB  LV  MB  LV  CV  AB  MB  MF  LV
   CB  MF  CB  MV  LF  CV  CB  AF  AV  AF
   AB  LB  LV  MF  LB  CB  CV  CB  CB  MF

   Organise this information in a two-way table.

4. Use your two-way table from Q3 to answer these questions.
   
   a. What percentage of the customers bought a mocha coffee and Bakewell cake?
   b. What fraction of the customers bought a latte coffee?
   c. Which coffee and cake combination was the most popular?

5. Susan asks 40 people how tall they are. Here are her results, in metres, to 2 d.p.

   1.69  1.45  1.99  1.93  1.66  1.72  1.61  1.74  1.66  1.55
   1.58  1.98  1.59  1.62  1.82  1.68  1.52  1.54  1.75  1.80
   1.95  1.63  1.84  1.71  1.48  1.81  1.49  1.87  1.67  1.88
   1.60  1.51  1.79  1.66  1.78  1.80  1.80  1.60  1.80  1.52

   Construct a grouped frequency table to show the heights of the 40 people.

6. The table shows the lengths of time that 100 patients had to wait to see a doctor in two different hospitals.

<table>
<thead>
<tr>
<th>Hours, h</th>
<th>0 ≤ h &lt; 1</th>
<th>1 ≤ h &lt; 2</th>
<th>2 ≤ h &lt; 3</th>
<th>3 ≤ h &lt; 4</th>
<th>4 ≤ h &lt; 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hospital A</td>
<td>56</td>
<td>22</td>
<td>12</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Hospital B</td>
<td>6</td>
<td>15</td>
<td>27</td>
<td>34</td>
<td>18</td>
</tr>
</tbody>
</table>

   Write a short report comparing patient’s waiting times in the two hospitals.
1. Work out these:  
   a. \(-3 \times -12 + 4\)  
   b. \(9 + 6 \times -2\)

2. In a survey, 10 waiters were asked how much they earn per hour. These are the results:
   £5.80 £5.20 £5.25 £5.85 £6.00 £6.05 £6.90 £5.60 £5.85 £6.10
   Find the mean wage per hour, using an assumed mean of £5.

3. Ashon and Reth are batsmen for the same cricket team. The table shows the range and the mean number of runs they score per match in one season.
   
<table>
<thead>
<tr>
<th>Range</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ashon</td>
<td>53</td>
</tr>
<tr>
<td>Reth</td>
<td>12</td>
</tr>
</tbody>
</table>

   Who do you think is the better batsman? Give a reason for your answer.

4. Lowri records the number of text messages she receives each day for two weeks. Here are her results:
   8 12 10 19 9 28 25 14 7 17 12 21 22 12
   Draw a stem-and-leaf diagram for this data. Remember to include a key.

5. Use your stem-and-leaf diagram from Q4 to answer these questions:
   a. What is the mode of the data?
   b. What is the range?
   c. What is the median?

6. The table shows the life span of some light bulbs tested by the manufacturer.
   
   | Life span, \(h\) (hours) | 160 \(\leq h\) \(< 180\) | 180 \(\leq h\) \(< 200\) | 200 \(\leq h\) \(< 220\) | 220 \(\leq h\) \(< 240\) | 240 \(\leq h\) \(< 260\) | 260 \(\leq h\) \(< 280\) |
   | Frequency          | 20   | 31   | 36   | 42   | 55   | 16   |

   a. How many light bulbs were tested?
   b. What is the modal class of this data?
   c. Re-group the data into 40-hour intervals (160 \(\leq h\) < 200, ...)
   d. What is the modal class now?

7. a. Rosine has four number cards.
   The range of the numbers on the cards is 6.
   The mode of the numbers on the cards is 8.
   What are the missing values on the number cards?

   b. Ryan has five number cards.
   The median of the numbers on the cards is 9.
   The mode of the numbers on the cards is 3.
   The range of the numbers on the cards is 10.
   The mean of the numbers on the cards is 8.
   What are the missing values on the number cards?
1. Rhys, Bryn and Dafydd share prize money of £4400 in the ratio 3:1:7. How much does each of them receive?

2. The table shows the colours of 20 teachers' cars. The frequencies of the colours can be shown by the angles of the sectors in a pie chart.

<table>
<thead>
<tr>
<th>Colour of car</th>
<th>Red</th>
<th>Blue</th>
<th>Silver</th>
<th>Black</th>
<th>Green</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>5</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Angle</td>
<td>72°</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>360°</td>
</tr>
</tbody>
</table>

   - Copy and complete the table.
   - Draw a pie chart to show the data.

3. The pie chart shows the results of a survey on how some pupils travel to school. 52 pupils walked to school.

   a. How many pupils were surveyed altogether?
   b. How many pupils travelled by bus?

4. The pie chart shows the results of a survey on favourite TV soap operas. Altogether 120 people were asked.

   a. How many people chose Eastenders as their favourite soap opera?
   b. How many people chose Neighbours as their favourite soap opera?

5. The pie charts show the sources of litter found on the beaches in Scotland and Wales during a beach litter survey.

   Use the pie charts to decide whether each statement is true or false, or whether there is not enough information to decide. Explain each of your answers.
   a. The fraction of litter from 'beach visitors' was bigger in Wales than in Scotland.
   b. Scotland had more items of litter from 'sewage' than Wales.
   c. The percentage of litter in Wales from 'other' was more than double the percentage of litter from 'other' in Scotland.
5.5 Line graphs

1. Work out these.
   a. \(24 \times \frac{1}{3}\)
   b. \(24 \times \frac{2}{3}\)
   c. \(63 \times \frac{1}{7}\)
   d. \(63 \times \frac{5}{7}\)

2. The table shows the population of a village, to the nearest 10 people, over a 40-year period.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>420</td>
<td>550</td>
<td>860</td>
<td>980</td>
<td>930</td>
</tr>
</tbody>
</table>

   a. Draw a line graph for the data.
   b. Use your graph to estimate the population of the village in 1985.

3. The line graph shows the profit made each year by a company over a ten-year period.

   a. How much profit did the company make in 2003?
   b. Between which two years did the company have the greatest increase in profits?
   c. Between which two years did the company have the greatest decrease in profits?

4. The graph shows the average prices of lamb meat in England and in Scotland in May and June of 2008.

   a. What was the difference in the price of lamb meat, in pence per kg, between England and Scotland on 1 May?
   b. On which day was the price of lamb meat the same in England and Scotland?
   c. On which day was there the greatest difference in the price of lamb meat in England and Scotland?
   d. If you were a sheep farmer in Scotland, on which day would it have been best to sell your lambs? Give a reason for your answer.
1. Which is the correct answer for each of these, A, B or C?
   a. 0.2 \times 0.1 A 0.2 B 0.02 C 0.002
   b. 0.03 \times 0.5 A 0.15 B 0.015 C 0.00015
   c. 1.3 \times 0.04 A 0.052 B 0.0052 C 0.000052
   d. 0.06 \times 0.05 A 0.3 B 0.03 C 0.003

2. Mr Byrd gave his Year 9 class a maths puzzle. These are the times, in seconds, that it took the pupils to complete the puzzle:
   48  32  22  18  26  25  33  37  39  28
   42  12  16  28  33  44  50  49  38  23
   9  19  15  21  42  47  46  31  28  17
   Draw a frequency diagram to show this data. Use five groups.

3. The frequency diagrams below show the life span of 100 light bulbs produced by two different companies, Ultralite and Britelite.
   Life span of 100 Ultralite bulbs
   Life span of 100 Britelite bulbs
   Which company produces longer lasting light bulbs? Use the frequency diagrams to justify your answer.

4. A health centre runs ‘give-up-smoking’ courses. This frequency polygon shows the ages of people attending the courses one week.
   a. How many people aged between 60 and 70 attended the course?
   b. How many people under the age of 40 attended the course?

5. A doctor carried out a survey on her patients one week. She asked them how long they had to wait to see her when they came into the surgery. These are her results.
<table>
<thead>
<tr>
<th>Time, m (minutes)</th>
<th>0 \leq m &lt; 10</th>
<th>10 \leq m &lt; 20</th>
<th>20 \leq m &lt; 30</th>
<th>30 \leq m &lt; 40</th>
<th>40 \leq m &lt; 50</th>
<th>50 \leq m &lt; 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>22</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>Afternoon</td>
<td>12</td>
<td>18</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
   a. Draw a frequency polygon for the morning data.
   b. On the same axes, draw a frequency polygon for the afternoon data.
   c. Compare the distribution of waiting times in the mornings and the afternoons.
1. Which of the numbers on the cards below are square numbers?

- 256
- 102
- 196
- 81
- 159
- 36
- 86
- 128
- 47
- 68
- 225
- 182

b. Write down the letters from the cards with square numbers on, starting with the smallest square number. What word have you written?

2. Convert each of these areas to square centimetres (cm²).
   a. 15 m²
   b. 0.1 m²
   c. 15 mm²
   d. 15 000 mm²

3. Write true or false for each of these.
   a. 5 cm² = 50 mm²
   b. 0.55 cm² = 55 mm²
   c. 500 cm² = 0.0005 m²
   d. 500 000 cm² = 5 m²

4. Convert 3500 cm³ to litres.

5. Write down the smallest and largest possible values for each measurement.
   a. 27 km
   b. 2.7 litres
   c. 0.27 mm

6. This is Hassam’s homework. Mark it for him and correct any mistakes.

   These quantities have been measured to the accuracy shown. Give the upper and lower bounds for each.
   a. 130 cm (to the nearest 10 cm)
   b. 8.3 mm (to 1 d.p.)
   c. 18 g (to the nearest gram)
   d. 407.88 ml (to 2 d.p.)
   e. 9300 t (to the nearest 100 tonnes)

   a. 125 cm ≤ x < 135 cm
   b. 8.20 mm ≤ x ≤ 8.40 mm
   c. 17.5 g < x < 18.49 g
   d. 407.8 ml ≤ x ≤ 407.9 ml
   e. 1250 t ≤ x < 1350 t

7. The length and width of this tile are given to the nearest mm.
   a. Work out the smallest and largest possible values for the length and the width of the tile.
   b. Work out the smallest possible value for the area of the tile.
1. In your head work out the value of this expression. \((-3 - 5)^2 - 100\)

2. Match each of the graphs on the left with the most likely description on the right.
   - a
   - b
   - c
   - A: A jogger running to the park and back again
   - B: A jogger running down a steep hill
   - C: A jogger running up a steep hill
   - D: A jogger running at a constant speed
   - E: A jogger running then stopping for a rest

3. The graphs show how the depth of water in three containers varies with time as water is poured into each container at a steady rate. Match each graph to the correct container.
   - a
   - b
   - c
   - A
   - B
   - C

4. The density of gold is 19.3 g/cm³. Work out the volume of 1 kg of gold. Give your answer correct to 2 decimal places.

5. Peter goes on a journey by car. The distance–time graph shows his journey.
   - a
     - i. During which part of the journey was Peter travelling at his fastest?
     - ii. Did he stop for a cup of tea?
   - b
     - Calculate the average speed for the whole journey.

6. Joe ran in a straight line for exactly 30 seconds. His GPS showed that he had run a distance of 220 metres, to the nearest metre. Work out Joe’s maximum possible speed.

7. The mass of a glass paperweight is 557.3 g correct to 1 decimal place. The volume of the glass paperweight is 124 cm³ correct to the nearest whole number. Work out the minimum possible density of the paperweight. Give your answer correct to 2 decimal places.
6.3 Circles and perimeter

1. Sam and Reth are lifeguards at different swimming pools. Sam is paid £5.90 per hour and is about to be given a 5% pay rise. Reth is paid £6.10 per hour and is about to be given a 2% pay rise.
   a. How much will Sam and Reth earn per hour after their pay rise?
   b. After their pay rise, who will earn the most and by how much?
   c. One week they both work for 30 hours. What is the difference in their earnings?

In Q2 to Q7 use the π button on your calculator.

2. A £2 coin has a radius of 14.2 mm. Calculate the circumference of the coin. Give your answer to 2 decimal places.

3. A £1 coin has a diameter of 22.5 mm. Calculate the circumference of the coin. Give your answer to 2 decimal places.

4. The circumference of a 10p coin is 77 mm. What is the radius of the coin? Give your answer to 1 decimal place.

5. Diagrams A and B show two different footpaths.

Which footpath is the shorter of the two, and by how many metres?

6. Calculate the length of the minor arc AB. Give your answer to 2 decimal places.

7. Calculate the total perimeter of this shape. Give your answer to 1 decimal place.
1. Write these negative decimal numbers in descending order.
   -2.5, -3.08, -2.19, -2.08, -3.89, -3.7

b. How many of the numbers in part a are less than -3.1?

In Q2 to Q7 use the $\pi$ button on your calculator.

2. A £2 coin has a radius of 14.2 mm. Calculate the area of one side of the coin. Give your answer to 2 decimal places.

3. A £1 coin has a diameter of 22.5 mm. Calculate the area of one side of the coin. Give your answer to 2 decimal places.

4. One side of a 5p coin has an area of 254.5 mm². What is the radius of the coin? Give your answer to the nearest mm.

5. John said that these two shapes have the same area. Is he correct? Explain your answer.

6. Calculate the area of the sector AOB. Give your answer to 2 decimal places.

7. Calculate the total area of this shape. Give your answer to 1 decimal place.
1. Shaun goes on holiday to Europe when the exchange rate is €1.15 = £1. Mei goes on holiday to Europe when the exchange rate is €1.08 = £1. They both exchange £200 into euros. How many more euros does Shaun receive than Mei?

2. Which two of these shapes have the same area?

3. Work out the area of this parallelogram. Give your answer to the nearest square centimetre.

4. Which two of these shapes have the same volume?

5. Calculate the surface area of cuboid C in Q4.

6. Use the formula $A = \frac{1}{2}h(a + b)$ to calculate the area of this trapezium.

7. Calculate the volume of this solid shape.

8. Calculate the surface area of the solid shape in Q7.
6.6 Volume and surface area

1. Calculate the exterior angle of a regular octagon.

2. a. Calculate the surface area of this cuboid.
   b. Calculate the volume of this shape.

3. Calculate the surface area of this shape.

4. Find the volume of each of these prisms.
   a. Area = \(24\) cm\(^2\)
   b. Area = \(7.2\) cm

5. A cuboid has a volume of \(126\) cm\(^3\).
The height of the cuboid is 4 cm and the width of the cuboid is 3.5 cm.
   a. Work out the length of the cuboid.
   b. Calculate the surface area of the cuboid.

6. Calculate the volume of this tin of paint.

7. This is Davin's homework. He has made one mistake.
   
   The volume of a cylinder is \(450\) cm\(^3\). The radius is 5.6 cm.
   Work out the height of the cylinder.
   
   | Area = \(\pi \times r^2 = \pi \times 5.6^2 = \pi \times 31.36 = 35.19 \text{ cm}^2\) |
   | Volume = Area \(\times\) Height |
   | \(450 = 35.19 \times \text{Height}\) |
   | Height = \(450 \div 35.19 = 12.79\) cm |
   
   Explain what Davin has done wrong and work out the correct answer for him.
1. Convert the following:
   a. 256 cm to m
   b. 26,500 cm$^3$ to litres
   c. 414 minutes to hours and minutes

2. Copy and complete this number crossword. The clues are listed below.
The answers to the red clues must be written in standard form.
The answers to the blue clues must be written as normal numbers.
The first one is done for you.

1 across: $7000 = 7 \times 10^3$

Across
1. What is 7000?
2. Change 2.3 \times 10^4 \text{mm} \text{ into metres.}
3. \((8.6 \times 10^{-2}) \div (2 \times 10^{-6})
4. \(3.2 \times 10^{-2} \times 100 \times 1000 \times 1000000
5. \text{What is } 12,000,000 \times 10^{-2} \times 10^{-4}?
6. \text{What is } 370000000?\)
7. \text{What is } 91,000 \times 10^{-4}?\)
8. \text{Which is the biggest number, } 0.04 \times 10^4, 40 \times 10^{-1} \text{ or } 4,000,000 \times 10^{-7}?\)
9. \text{What is } 3.7 \times 10^{-3}?\)
10. \((8.8 \times 10^2) \times (5 \times 10^4)\)

Down
2. Change 200 m into km.
3. \((9.5 \times 10^3) \div (4.8 \times 10^4)
4. \text{Which is the smallest number, } 3010 \times 10^3, 0.003 \text{ or } 10 \times 10^{-6}\)
5. \text{What is } 2000 \times 2000\)
6. \text{What is } 0.0000003 \times 10^5?\)
7. \((3.1 \times 10^9) + (7 \times 10^3)\)
8. \text{What is } 2,000,000,000?\)
9. \((4.125 \times 10^3) - 125\)
10. \((12 \text{ down}) \times 10^{-6}\)
11. \((1 \text{ across}) \div 10\)
7.2 To round or not to round?

1. Work out these.
   a) \( \frac{3}{5} \times 21\) g
   b) \( \frac{7}{8} \times £96\)
   c) \( \frac{2}{3} \times £475\)
   d) \( \frac{11}{20} \times 320\) kg
   e) Work out 'your answer to part b' ÷ 4.
   f) Work out 'your answer to part c' - 'your answer to part e'.
   g) What is special about the number you found in part f?

2. Write true or false for each of these.
   a) \( 0.\overline{2} = \frac{22}{100}\)
   b) \( 0.6\overline{3} = \frac{7}{11}\)
   c) \( 0.\overline{3} = \frac{3}{11}\)

3. Copy and complete the table.

<table>
<thead>
<tr>
<th></th>
<th>to 1 d.p.</th>
<th>to 3 d.p.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>36.7994</td>
<td>36.8</td>
</tr>
<tr>
<td>b</td>
<td>29.4567</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>29.9876</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>0.3132</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>0.07777</td>
<td></td>
</tr>
</tbody>
</table>

4. Copy and complete the table.

<table>
<thead>
<tr>
<th></th>
<th>Accurate answer</th>
<th>Answer to an appropriate degree of accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>6.32 × 8.11</td>
<td>51.2552</td>
</tr>
<tr>
<td>b</td>
<td>8.11 ÷ 6.32</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>£37.97 ÷ 5</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>12 cm ÷ 7</td>
<td></td>
</tr>
</tbody>
</table>

5. A builder uses this formula to work out the amount of wooden flooring needed for a room.

Wooden flooring needed (m²) = area of floor (m²) × 1.1

The rectangular floor of a kitchen measures 4.06 m by 3.88 m.
   a) By rounding the measurements to the nearest whole number, estimate the amount of wooden flooring needed for this kitchen.
   b) Use a calculator to work out the exact amount of wooden flooring needed for the kitchen.
   c) Do you think that rounding to the nearest whole number is a sensible way to estimate in this case?

6. Use an algebraic method to convert each recurring decimal to a fraction.
   a) \( 0.\overline{5} \)
   b) \( 0.\overline{52} \)
   c) \( 0.\overline{247} \)

7. a) Use an algebraic method to convert 0.\(\overline{4}\) to a fraction.
   b) What do you notice about your answer to part a? Can you explain this?
7.3 Using a calculator

1. In this number wheel, opposite numbers add to make -8. Copy the wheel and fill in the missing numbers.

2. a Match each red card with the correct yellow card.
   - $\frac{33}{40}$
   - $\frac{3}{40} \times \frac{11}{40}$
   - $\frac{5}{40}$ or $\frac{1}{8}$
   - $\frac{1}{5} \times \frac{5}{8}$
   - $\frac{5}{40}$ or $\frac{3}{20}$
   - $\frac{33}{1600}$

   b Which card hasn't been used?

3. Use a calculator to work out these.
   a $3 \frac{1}{2} + 2 \frac{3}{5}$
   b $3 \frac{1}{2} - 2 \frac{3}{5}$
   c $3 \frac{1}{2} \times 2 \frac{3}{5}$
   d Write down the button presses needed on your calculator to answer part c.

4. Use your calculator to work out $365 \times 1$ hour 14 minutes. Give your answer in hours and minutes.

5. Find the reciprocal of each of these.
   a 0.4
   b 8
   c $\frac{5}{6}$
   d $3 \frac{1}{2}$
   e Write down the button presses needed on your calculator to answer part d.

6. a Use the reciprocal key on your calculator to work out $\frac{1}{3^2 \times 7.5 \times 3}$
   b Write down the button presses needed on your calculator to answer part a.

7. a Use your calculator to find the reciprocal of 16.
   b Without using your calculator, find the reciprocal of 0.0625.
7.4 More rounding

1. Use the fact that $27.5 \times 36 = 990$ to work out these.
   a. $2.75 \times 3.6$
   b. $275 \times 0.36$
   c. $27.5 \times 18$

2. Copy and complete.
   a. $2,968,000 = \underline{\text{_____}}$ million
   b. $207,800 = \underline{\text{_____}}$ thousand
   c. $\underline{\text{_____}} = 2.4$ thousand
   d. $\underline{\text{_____}} = 0.8$ million

3. Here is some information about Wales.

<table>
<thead>
<tr>
<th>Population</th>
<th>2.9 million</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land area</td>
<td>20,781 million m²</td>
</tr>
<tr>
<td>Number of cars</td>
<td>2.4 million</td>
</tr>
<tr>
<td>Number of people</td>
<td>1.64 million</td>
</tr>
<tr>
<td>with a driving licence</td>
<td></td>
</tr>
</tbody>
</table>

   a. Estimate the mean number of cars per person in Wales.
   b. Estimate the mean number of cars per person with a driving licence in Wales.
   c. Estimate the mean area of land per person in Wales.
   d. Estimate the mean area of land per car in Wales.

4. A school hall has a capacity of 410 people. Given that tickets for ‘Stars in their eyes’ cost £3.95 each, estimate the total revenue from ticket sales when the hall is full.

5. This is Jeff’s homework on rounding. He has made three mistakes.

<table>
<thead>
<tr>
<th>Round these numbers to the given number of significant figures.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $870$ (1 s.f.) = $400$</td>
</tr>
<tr>
<td>2. $0.37$ (1 s.f.) = $0.40$</td>
</tr>
<tr>
<td>3. $890$ (2 s.f.) = $8900$</td>
</tr>
<tr>
<td>4. $0.08900$ (2 s.f.) = $0.09$</td>
</tr>
<tr>
<td>5. $4.1414$ (3 s.f.) = $4.14$</td>
</tr>
<tr>
<td>6. $4.1414$ (3 s.f.) = $4.14$</td>
</tr>
<tr>
<td>7. $9.99$ (1 s.f.) = $100$</td>
</tr>
<tr>
<td>8. $1.9999$ (4 s.f.) = $2.0000$</td>
</tr>
<tr>
<td>9. $0.0000109$ (1 s.f.) = $0.00001$</td>
</tr>
<tr>
<td>10. $10.91$ (2 s.f.) = $11$</td>
</tr>
</tbody>
</table>

   Find the mistakes and correct them.

6. A company estimates that its average pay per employee is £26,000, to the nearest £1000.
   a. What is the largest possible value for the average pay per employee?
   b. What is the smallest possible value for the average pay per employee?
   c. Use inequality signs to identify the upper and lower limits of the possible average pay.

7. A company director earns £260,000, to the nearest £10,000.
   Her pay is spread equally over the year.
   Work out the upper and lower bounds of the director’s monthly pay.
7.5 Roots and standard form on a calculator

1. Write these lengths in order, smallest first.
   - 20 cm
   - 2 m
   - 0.25 m
   - 0.02 km
   - 25 mm
   - 520 mm
   - 250 m

2. Copy the table below. Link the square root on the left with the estimate in the middle with the accurate answer on the right. The first one is done for you.

<table>
<thead>
<tr>
<th>Estimate of square root without a calculator</th>
<th>Accurate value of square root with a calculator, to 2 d.p.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a  $\sqrt{52}$</td>
<td>7</td>
</tr>
<tr>
<td>b  $\sqrt{49}$</td>
<td>7</td>
</tr>
<tr>
<td>c  $\sqrt{67}$</td>
<td>8</td>
</tr>
<tr>
<td>d  $\sqrt{21}$</td>
<td>8</td>
</tr>
<tr>
<td>e  $\sqrt{33.5}$</td>
<td>7</td>
</tr>
<tr>
<td>f  $\sqrt{49.98}$</td>
<td>5</td>
</tr>
</tbody>
</table>

3. Use your calculator to work out these.
   - $a \sqrt{4096}$
   - $b \sqrt{941.92}$
   - $c \sqrt{0.009261}$
   - $d \sqrt{343}$

4. Use your calculator to work out these roots to one decimal place.
   All the answers are in the cloud.
   - $i \sqrt{1296}$
   - $ii \sqrt{2187}$
   - $iii \sqrt{0.0000256}$
   - $iv \sqrt{0.000074088}$
   - $b$ Which numbers from the cloud haven't you used?
   - $c$ Work out the fifth root of both of these numbers. What do you notice about your two answers?

5. Use your calculator to work out these.
   Give your answers in standard form to one decimal place.
   - $a 2.45 \times (3.9 \times 10^2)$
   - $b 12 \div (6.82 \times 10^{-5})$
   - $c (4.78 \times 10^7) \times (8.2 \times 10^{-21})$
   - $d (6.5 \times 10^{-3}) \div (4.2 \times 10^9)$

6. In 2007, approximately 68.07 million passengers passed through Heathrow airport. Each passenger carried on average 21.4 kg of luggage. What is the total weight of luggage that passed through Heathrow airport in 2007? Give your answer in standard form correct to one decimal place.

7. The mass of the Earth is $5.97 \times 10^{24}$ kg. The mass of the Moon is $7.35 \times 10^{22}$ kg. How many times heavier is the Earth than the Moon? Give your answer correct to one decimal place.
1. Work out these.
   a) \( \frac{1}{2} \) of 54 m
   b) \( \frac{1}{3} \) of £64
   c) \( \frac{1}{2} \) of 13 cm
   d) \( \frac{3}{5} \) of 240 m²

2. Write true or false for each of these.
   a) \( 0.3 \times 0.3 = 0.9 \)
   b) \( 0.02 \times 0.5 = 0.01 \)
   c) \( 0.7 \times 0.7 = 0.49 \)
   d) \( 0.9 \times 0.002 = 0.00018 \)
   e) \( 0.8 \times 0.8 = 0.064 \)
   f) \( 0.001 \times 0.01 = 0.00001 \)
   g) \( 0.2 \times 0.2 \times 0.2 = 0.8 \)
   h) \( 0.3 \times 0.2 \times 0.01 = 0.0006 \)

3. Use a written method to work out
   \[ 1.78 \times 5.8 \]

4. Work out these without a calculator.
   a) \( 0.08 \div 0.2 \)
   b) \( 0.8 \div 0.02 \)
   c) \( 5.22 \div 1.2 \)
   d) \( 23.5 \div 0.25 \)

5. Work out these without a calculator.
   a) \( 1.73 + 0.173 + 173 + 0.00173 \)
   b) \( 17.3 - 0.0173 - 1.73 \)
   c) \( 17.3 - 1.73 + 0.0173 \)
   d) \( -17.3 - 173 - 0.0173 \)

6. Jasmine buys some saffron for cooking.
   Saffron costs £3.55 per gram.
   Jasmine spends £18.
   How many grams of saffron does she buy?
   Give your answer correct to one decimal place.

7. Work out
   \[ 0.0246 \div 0.26 \]
   Give your answer to a suitable number of significant figures.
7.7 Using a calculator efficiently

1. Use the facts that \(1\text{ cm}^2 = 100\text{ mm}^2\) and \(1\text{ m}^2 = 10 000\text{ cm}^2\) to copy and complete these.
   - a. \(9\text{ cm}^2 = \) ________ \(\text{ mm}^2\)
   - b. \(1.5\text{ cm}^2 = \) ________ \(\text{ mm}^2\)
   - c. ________ \(\text{ cm}^2 = 2400\) \(\text{ mm}^2\)
   - d. \(8\text{ m}^3 = \) ________ \(\text{ cm}^3\)
   - e. \(0.7\text{ m}^2 = \) ________ \(\text{ cm}^2\)
   - f. ________ \(\text{ m}^2 = 240 000\) \(\text{ cm}^2\)

2. Use the brackets or memory keys on your calculator to work out these. Give your answers to two decimal places.
   - a. \(3.6 + 4.97\) \(\frac{8.81 - 5.6}{5.1 \times 1.5^3}\)
   - b. \((3.7 + 2.05)^2\) \(\frac{50 - 4.77}{336 \times 0.2^2}\)

3. Which is the correct answer, to one decimal place, for each of these, A, B or C?
   - a. \((34.76 - 8.9 \times 3.5)^3\) A -346.8 B 13.0 C 47.0
   - b. \(\sqrt{2.5 \times 6.4 - 3.45}\) A 2.7 B 3.5 C 6.7
   - c. \(\left(\frac{4}{5}\right)^2 \times \sqrt{6.7 + 8.5}\) A 1.9 B 9.4 C 19.7
   - d. \(4.67^2 - (3.62 \div 0.7)^3\) A -116.5 B 11.3 C 53.0

4. In a sale, Delyth buys two skirts priced at £18.50 each and three blouses priced at £12.99 each. The total price is reduced by 30% at the checkout.
   - a. Write down one way of calculating the final price with a single calculation.
   - b. Work out the final price using a calculator.

5. Write the answers to these calculations in order of size, starting with the smallest.
   - \(\sqrt[3]{4.76 \times 18.94} = \frac{1.76^5 + 0.23}{4.75 \times 3.79} = \frac{1}{(5.8^2 - \sqrt{20})^3}\)

6. Use the memory key on your calculator to help you solve this problem.
   Pete, Gavin and Kirish share a lottery win of £13 850 in the ratio 2 : 3 : 4. Kirish puts \(\frac{3}{4}\) of his money into a savings account that earns him 2.72% interest each year. How much will Kirish have in the bank at the end of one year?

7. This diagram shows the dimensions of a football pitch \(ABCD\).
   In football training the coach makes the players run two circuits from \(A\) to \(B\) to \(C\) to \(D\) to \(A\).
   - a. Work out the length of the diagonal \(AD\).
   - b. What is the total distance the players run in these two circuits?
     Give your answer to the nearest metre.
1. Use 10, 100, 1000, x, +, and = to form four correct calculations, using the numbers from the cloud. For example, \(3.5 \times 100 = 350\).

2. The difference between two whole numbers is 130. One of the numbers is three times the size of the other number.
   a. Write three different pairs of whole numbers whose difference is 130.
   b. Test the pairs from part a to see if one of the numbers is three times the other.
   c. If you haven’t found the correct pair of numbers yet, try some other pairs.

3. I think of a number \(x\), subtract 7 and multiply the result by 5. The answer is 80. Form an equation and solve it to find the value of \(x\).

4. A triangular plot of land has a base length of \(x\) m. The perpendicular height of the triangular plot is 26.5 m more than the base length. The area of the plot is 1246 m\(^2\). Form an equation and use trial and improvement to find the value of \(x\) to one decimal place.

5. Ian is a graphic designer. He is designing the front of a cereal box. The front of the box measures 19 cm by 28 cm. He must have the company logo inside a circle that uses 20% of the area of the front of the box. What must be the radius of the circle? Give your answer to the nearest mm.

6. I think of a number \(x\). I add 3, square the result and finally subtract 15. The answer is 274.
   a. Write down an equation in \(x\).
   b. Solve the equation to find the value of \(x\).

7. A garden path is 18 m long, to the nearest metre.
   a. What are the shortest and longest possible lengths of the path?
   A paving slab is 50 cm long, to the nearest 10 cm.
   b. What are the shortest and longest possible lengths of the paving slab?
   c. The path will be one paving slab wide. What are the smallest and greatest numbers of paving slabs that will be needed to pave the path?
1. Match each red card with the correct yellow card.

a) \( \frac{11}{24} \)  
   b) \( \frac{2}{9} \times \frac{3}{16} \)  
   c) \( \frac{25}{36} \)  
   d) \( \frac{1}{8} + \frac{9}{20} \)  
   e) \( \frac{3}{4} \)  
   f) \( \frac{1}{3} + \frac{5}{12} \)  
   g) \( \frac{23}{40} \)  
   h) \( \frac{7}{9} - \frac{1}{12} \)  
   i) \( \frac{1}{24} \)  

b) Which card hasn’t been used?

c) Write a fraction multiplication card to go with this answer card.

2. Find the highest common factor (HCF) of 12, 60 and 78.

3. Find the lowest common multiple (LCM) of 12, 16 and 36.

4. Jared has spilt his tea on his homework!
   Work out the numbers that have been hidden by the tea stain.

   Find the prime factor decomposition of each of these numbers.
   Write down your answers as powers.

   Q1. 128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7
   Q2. 100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2
   Q3. 98 = 2 \times 7 \times 7 = 2 \times 7^2
   Q4. 441 = 3 \times 3 \times 3 \times 7 = 3^2 \times 7
   Q5. 108 = 2 \times 2 \times 3 \times 3 \times 3 = 2 \times 3^3

5. Mark has two bulbs, A and B, that turn on and off in a regular pattern. Bulb A turns on, stays on for 4 s and then turns off and stays off for 4 s. Bulb B turns on, stays on for 7 s and then turns off and stays off for 7 s. Mark turns both bulbs on at the same time. How long will it be before both bulbs turn on again at the same time?

6. Alun, Bradley and Collister are counting drumbeats. Alun hits a kettle drum every two beats. Bradley hits a snare drum every five beats. Collister hits a bass drum every eight beats. Alun, Bradley and Collister start hitting their drums at the same time. How many beats is it before they next hit their drums at the same time?

7. Use prime factor decomposition to find the HCF and LCM of 15, 35 and 50.
8.2 Using factors and multiples

1. In this number wheel, opposite numbers add to make 1. Copy the wheel and fill in the missing numbers.

2. Using only the \( \sqrt{} \) key on your calculator, estimate the square root of 2927. Give your answer to one decimal place.

3. Using only the \( \times \) or \( \times \) key on your calculator, estimate the cube root of 2927. Give your answer to one decimal place.

4. Do not use a calculator for this question. Use factors to work out the square root of 784.

5. Do not use a calculator for this question. Use factors to work out the cube root of 729.

6. Write these as simply as possible in surd form.
   \[ a \sqrt{24} \quad b \sqrt{54} \quad c \sqrt{20} \quad d \sqrt{32} \]

7. Ellie has one question to do for homework. It is quite a hard question, so she has four attempts at it. Which attempt gives the correct answer?

   Question:
   Work out the perimeter of this rectangle.
   \( \sqrt{7} + 2 \)

   1st attempt: \( \sqrt{7} + 2 + \sqrt{7} - 2 = 2\sqrt{7} \)

   2nd attempt: \( (\sqrt{7} + 2)(\sqrt{7} - 2) = \sqrt{7} - 2\sqrt{7} + 2\sqrt{7} - 4 = 3 \)

   3rd attempt: \( \sqrt{7} + \sqrt{7} + \sqrt{7} + \sqrt{7} = 4\sqrt{7} \)

   4th attempt: \( 2 \times (\sqrt{7} + 2 - \sqrt{7} + 2) = 2 \times 4 = 8 \)
8.3 Multiplying and dividing with indices

1. Copy these numbers. They are a coded message!

<table>
<thead>
<tr>
<th></th>
<th>8</th>
<th>-8</th>
<th>-5</th>
<th>-6</th>
<th>6</th>
<th>-15</th>
<th>4</th>
<th>6</th>
<th>-2</th>
<th>2</th>
<th>-4</th>
<th>2</th>
<th>15</th>
<th>-5</th>
</tr>
</thead>
</table>

b. Each number in the code is the answer to one of the calculations in the key. Replace each number with the matching letter to crack the code. The first one has been done for you.

\[-5 \times 3 = -15\]

So -15 represents U.

What is the secret message?

Key:
-5 \times 3 = U
20 \div -4 = E
-2 \times -1 = I
-18 \div 3 = Y
-3 \times -2 = O
-30 \div -2 = V
16 \div -8 = S
-3 - 5 + 4 = T
-2 \times (7 - 11) = A
12 \times -2 + 7 \times 4 = P
-30 + 5 - 2 = R

2. Simplify each expression. Leave your answer as a power.
   a. \(2^3 \times 2^5\)
   b. \(3^3 \times 3^7\)
   c. \(5^7 \div 5^2\)
   d. \(7^2 + 7^3 \times 7^5\)

3. Simplify each expression. Leave your answer as a power.
   a. \(\frac{2^3 \times 2^5}{2^7}\)
   b. \(\frac{3^3 \div 3^3}{3^7}\)
   c. \(\frac{5^3 \times 5}{5^2 \times 5^2}\)

4. Simplify each expression. Leave your answer as a power.
   a. \(a^2 \times a^3\)
   b. \(b^3 \div b\)
   c. \(c^{16} \times c^2\)
   d. \(d^{25} + d^{50}\)

5. Write true or false for each of these.
   a. \(b^3 \times b^3 = 2(b^3)\)
   b. \(c^3 \times c^3 = c^6\)
   c. \(2a^2 \times 3a^3 = 6a^5\)
   d. \(10a^3 \div 2a = 5a\)
   e. \(3f^2 \times f = 9f^2\)
   f. \(2g^2 \times 2g^2 \times 2g^2 = 8g^6\)

6. Work out the value of each expression.
   a. \(3^0\)
   b. \(64^0\)
   c. \(100^0 \times 3^2\)
   d. \(5^3 + 5^3 \times 4^2\)
   e. \(a^4 \div a^4\)
   f. \(6b^2 + 2b^2\)

7. Simplify each expression. Leave your answer as a power.
   a. \((2^3)^4\)
   b. \((2^4)^3\)
   c. \((4^2)^3\)
   d. \((4^3)^2\)
   e. \((4a^3)^2\)
   f. \((4a^2)^3\)
8.4 Index laws with negative and fractional powers

1. Here is some information about Scotland.

- Population of Scotland: 5.2 million
- Population of Edinburgh: 700,000
- Land area: 79,000 km²
- Number of islands: 780
- Number of islands that are inhabited: 130

   a. Estimate the mean number of people per km² in Scotland.
   b. What percentage of the population of Scotland lives in Edinburgh? Give your answer to one decimal place.
   c. What fraction of the Scottish islands are inhabited? Give your answer in its lowest terms.

2. Simplify each expression. Leave your answer as a power.
   a. \( a^2 + a^3 \)
   b. \( a^5 + a^6 \)
   c. \( a^3 + a^5 \)
   d. \( a^3 + a^6 \)

3. Find the value of each expression without using a calculator.
   Give your answer as a fraction.
   a. \( 2^{-1} \)
   b. \( 10^{-6} \)
   c. \( 5^{-3} \)
   d. \( 9^{-2} \)

4. Some of these expressions have the same value. Work out which ones do not.

\[
\begin{align*}
8^\frac{6}{3} & \quad (8^0)^2 \\
(2^{12})^{\frac{1}{2}} & \quad (8^3)^{\frac{3}{2}} \\
(22.5)^2 & \quad 16^{\frac{3}{2}} \\
4^{\frac{6}{2}} & \quad (8^{\frac{4}{3}})^{\frac{3}{2}}
\end{align*}
\]

5. Simplify each expression. Leave your answer as a fraction.
   a. \( 8^{-\frac{1}{2}} \)
   b. \( 125^{-\frac{3}{2}} \)
   c. \( (e^3)^{-2} \)
   d. \( d^2 + d^0 \)

6. Simplify each expression.
   a. \( (a^3)^2 \)
   b. \( (b^6)^{\frac{1}{3}} \)
   c. \( c^3 + c^3 \)
   d. \( d^3 + d^{4\frac{1}{2}} \)

7. Simplify each expression.
   a. \( a^{-3} \times a^3 \)
   b. \( b^{-4} \times b^{-\frac{1}{2}} \)
   c. \( c^{-4} + c^{-\frac{1}{2}} \)
   d. \( \frac{1}{16^{-\frac{1}{2}}} \)
1. Which is the correct answer, to one decimal place, for each of these, A, B or C?
   a. \((0.76 + 0.19 \times 4.1)^3\)   A 13.9   B 4.6   C 3.6
   b. \(\sqrt[4]{1.4 \times 7.3 + 0.45}\)   A 3.3   B 6.3   C 9.1
   c. \(\left(\frac{2}{3}\right)^2 \times \sqrt[3]{8.9 - 3.78}\)   A 0.8   B 2.3   C -0.8
   d. \(2.7^3 - (18.2 / 11.8)^2\)   A 19.6   B 17.3   C 5.7

2. Which of these points lie on the line \(y = 3x - 2\)?
   A (2, 4)   B (4, 2)   C (3, 7)   D (0, -2)   E (-2, -8)

3. a. Copy and complete this table of values for the straight-line graph \(y = 3x + 2\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y = 3x + 2)</td>
<td>-7</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Plot the graph on a coordinate grid with \(x\) values from -3 to +2 and \(y\) values from -7 to +8. Join the points to give a straight line.

4. Plot these straight-line graphs on a coordinate grid with \(x\) values from -4 to +4 and \(y\) values from -10 to +10.
   a. \(y = 2x + 1\)   b. \(y = -1 - 2x\)

5. These cards all have equations of straight lines on them.

   A. \(y = \frac{1}{2}x + 2\)  
   B. \(y = x + 2\)  
   C. \(y = 4x - 2\)
   D. \(y = 2x - 6\)  
   E. \(y = 2 - 3x\)  
   F. \(y = 3x + 3\)

   a. Which card has the equation of the steepest line?
   b. Which cards have the equation of a line that passes through (0, 2)?
   c. Which card has the equation of a line that passes through (0, 3) and has gradient 3?

6. Rearrange these equations of straight lines into the form \(y = mx + c\).
   a. \(y + 2 = x\)   b. \(y - 2x = 6\)   c. \(2x - 2y = 8\)   d. \(0 = 3 - y - x\)

7. Match each of these equations of straight lines with a parallel line from Q5.
   a. \(y - 4x = 7\)  b. \(y = 3x\)  c. \(2y = x + 4\)
   d. \(y = x - 20\)  e. \(0 = y - 2x + 2\)  f. \(3x = 2 - y\)
8.6 Investigating the properties of straight-line graphs

1. Without using a calculator, work out these.
   a) $0.09 \div 0.3$
   b) $0.16 \div 0.02$
   c) $9.1 \div 1.3$

2. a) Which of these lines are parallel to one other?
   b) Which of these lines are perpendicular to one other?

3. Use the grid in Q2 to answer these questions.
   a) Find the gradient of the lines B, C and E.
   b) Write down the coordinates of the point where the lines B, C and E cross the y-axis.
   c) Write down the equations of the lines B, C and E.

4. Here is a table of values of a linear function.
   Write down the equation of the line in the form $y = mx + c$.

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-10</td>
<td>-7</td>
<td>-4</td>
<td>-1</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

5. The yellow cards show the equations of straight lines.
   The blue cards show the coordinates of the points where the lines cross the y-axis.
   a) Match each yellow card to the correct blue card.
   b) Which two cards are left over?

   - $y = 7x - 3$
   - $y = -2x + 4$
   - $y = \frac{1}{2}x + 2$
   - $2y = 4x - 4$
   - $y = 2x$
   - $3x + y = -1$

   - $(0, -4)$
   - $(0, -1)$
   - $(0, 0)$
   - $(0, 2)$
   - $(0, -3)$
   - $(0, \frac{1}{2})$
   - $(0, -2)$

6. By plotting the intercept on the y-axis and then plotting another point by moving 1 unit across and $m$ units up or down, plot the graph of $y = -3x + 6$.

7. Here are the equations of six straight lines.

   A: $y = -2x + 3$
   B: $y = -\frac{1}{2}x - 3$
   C: $y = -2x - 3$
   D: $y = \frac{1}{2}x + 3$
   E: $y = 2x - 3$
   F: $y = \frac{1}{2}x - 3$

   Choose one of the equations to complete each of these statements.
   a) $y = 2x + 3$ is parallel to line _____.
   b) $y = 2x + 3$ is perpendicular to line _____.
8.7 Graphs of quadratic and cubic functions

1. Work out $3.25 \times 4.74$. Give your answer to an appropriate degree of accuracy.

2. Here is a table of values for the quadratic function $y = 4x^2$.
   - Copy and complete the table.
   - Draw a coordinate grid with $x$ values from -3 to 3 and $y$ values from 0 to 45.
   - Plot and draw the graph of $y = 4x^2$ using your table of values.

   \[ \begin{array}{c|c|c|c|c|c|c|c}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   \hline
   y = 4x^2 & 36 & 16 & \quad & \quad & \quad & \quad & 36 \\
   \end{array} \]

3. Here is a table of values for the quadratic function $y = 4x^2 + 5$.
   - Copy and complete the table.
   - On your coordinate grid from Q2, plot and draw the graph of $y = 4x^2 + 5$ using your table of values.

   \[ \begin{array}{c|c|c|c|c|c|c|c}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   \hline
   4x^2 & 36 & 16 & \quad & \quad & \quad & \quad & 36 \\
   +5 & \quad & +5 & +5 & +5 & +5 & \quad & +5 \\
   y = 4x^2 + 5 & 41 & 21 & \quad & \quad & \quad & \quad & 41 \\
   \end{array} \]

4. Here is a table of values for the cubic function $y = 4x^3$.
   - Copy and complete the table.
   - Draw a coordinate grid with $x$ values from -2 to 2 and $y$ values from -32 to 32.
   - Plot and draw the graph of $y = 4x^3$ using your table of values.

   \[ \begin{array}{c|c|c|c|c|c|c}
   x & -2 & -1 & 0 & 1 & 2 \\
   \hline
   y = 4x^3 & -32 & \quad & 0 & 32 & \quad \\
   \end{array} \]

5. Match each of these functions to the correct graph.
   - $y = x^2 + 20$
   - $y = -x^2 + 10$
   - $y = 10x^2$
   - $y = -10x^2$
   - $y = 10x^2 - 40$
   - $y = 2x^3$

6. a. For each curve on the grid in Q5, state whether it has a maximum or a minimum. Give the coordinates of any maximum or minimum.
   b. Write down the equations of any lines of symmetry of the curves.
8.8 Functions and graphs in real life

1. Round 0.058271 to three decimal places.

2. Write a formula for each of these.
   a. The cost \( C \), in pounds, of \( p \) pens costing 75p each.
   b. The cost \( C \) of buying \( c \) calculators costing £5 each, with a delivery charge of £3 regardless of the number of calculators bought.

3. The sketch shows the 40-minute journeys to work made by Hannah and Anna. Hannah travels by car and Anna travels by bicycle. Describe their journeys.

4. These graphs show how the height of water in a container changes with time, as the water is steadily poured into it.

Match each of these containers to the correct graph.

- a
- b
- c
- d

5. For the situation in part b of Q2, plot and draw an accurate graph to show the relationship between \( C \) and \( c \). Use values for \( c \) ranging from 0 to 10 calculators.

6. Naomi wants to find the best car hire deal, out of these three companies.

   - Super car hire: £10 per day plus 50p per mile
   - Sensational car hire: £25 per day plus 20p per mile
   - Stupendous car hire: £50 per day and no mileage charge

   a. Work out a formula for each car hire company.
   b. Draw graphs for each car hire company on the same grid.
   c. Naomi wants to hire a car for one day to travel 100 miles. Which company should she use?
   d. Explain the significance of the points where the line from one car hire company crosses the line for another car hire company.
8.9 Investigating patterns

1. Write true or false for each of these.
   \[ a \ 0.4 = \frac{44}{100} \]
   \[ b \ 0.72 = \frac{8}{11} \]
   \[ c \ 0.8 = \frac{81}{90} \]
   \[ d \ 0.3 = \frac{1}{3} \]
   \[ e \ 0.41 = \frac{41}{100} \]
   \[ f \ 0.05 = \frac{1}{20} \]

Q2 and Q3 use this sequence of shapes made from dots:

- Shape 1: 6 dots
- Shape 2: 10 dots
- Shape 3: 14 dots

2. Find the next two terms of the sequence above.

3. Find the \( n \)th term of the sequence above.

4. A T-shape is placed on a 10 by 10 number grid.
   The T-number is the number at the bottom of the T.
   The T-total is the sum of all the numbers in the T.
   In this case the T-number is 23 and the T-total is 258.
   a. Find the T-total for the T-numbers 24 and 25.
   b. Find an expression for the T-total for the T-number \( n \).

5. In a tech project, Terry has a piece of paper measuring 30 cm by 20 cm.
   By cutting squares from each corner and then folding the paper, he must make the box
   with the largest possible volume.
   He is only allowed to cut out squares from the corners which are whole numbers of
   centimetres long.
   For example:

   a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Side length of square to be cut out (cm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of box (cm³)</td>
<td>832</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Which size of cut-out square makes the box with the largest possible volume?
   c. Use a graph to prove your answer.

6. Explain algebraically how you can show that your expression from part b of Q4 is true
   for any T-shape in a 10 by 10 grid.
9.1 Fair play

1. Match each blue card with the correct yellow card.
   - \((2 \times 6 - 8)^2\)
   - \(53 - 42 \div 6\)
   - \(15 - 12 \div 4 + 8^2\)
   - \((7 - 2)^2 + 22\)
   - \(6^2 - (12 - 3)\)
   - \(46\)
   - \(47\)
   - \(16\)

2. Alun has two boxes of chocolates.
   The probability of getting a hard centred chocolate from box A is \(\frac{12}{20}\).
   The probability of getting a hard centred chocolate from box B is \(\frac{12}{24}\).
   Which box should Alun pick from to have the best chance of getting a hard centred chocolate? Explain your answer.

3. On Halloween Bim and Carol give toffees to children playing ‘Trick or treat’.
   Bim has 100 toffees, 10 of which have mustard inside!
   Carol has 50 toffees, 10 of hers also have mustard inside!
   If you were playing ‘Trick or treat’, would you have a higher chance of getting a mustard toffee from Bim or from Carol?

4. Li has two bags of sweets.
   The probability of getting a red sweet from bag A is \(\frac{6}{20}\).
   The probability of getting a red sweet from bag B is \(\frac{8}{25}\).
   Which bag should Li pick from to have the best chance of getting a red sweet? Explain your answer.

5. A group of children are going to race their bikes around a circuit.
   Which of these conditions are fair?
   A. Everyone sets off at the same time.
   B. The youngest child has a two-second head-start.
   C. Everyone goes in turn and their time is recorded.
   D. If there is a draw, the oldest child wins.

6. Three coins are dropped. Each coin can land either heads (H) or tails (T).
   a. Write down all the possible outcomes.
   b. What is the probability of getting exactly two heads?
   c. What is the probability of getting at least two tails?

7. Alice and Joe take it in turns to roll two dice and add the scores.
   a. Write down the possible outcomes and their probabilities.
   b. Before they roll the dice, they guess the total score.
   What score would you guess? Explain your answer.
1. Insert the correct sign, <, > or =, between each pair.
   a. $43 \div 0.1$ [ ] $430 \times 0.1$
   b. $70 \times 0.01$ [ ] $0.7 \div 0.1$
   c. $45 \div 0.1$ [ ] $0.45 \div 0.1$
   d. $30 \times 0.1$ [ ] $0.003 \div 0.001$

2. Which of these outcomes are mutually exclusive?
   Picking a student at random and then being:
   A. a boy or a girl
   B. a rugby player or a basketball player
   C. someone who likes maths or who likes PE
   D. someone who is the same height as you or is taller than you?

3. Keira has this spinner.
   She spins the spinner two times.
   a. Write down all the possible outcomes for the two spins.
   b. What is the probability of each outcome if it doesn’t matter what order the numbers are in?
   c. Add up all of your probabilities from part b.
      Are you surprised by your answer? Explain.

4. A game involves spinning two identical spinners.
   A player spins both the spinners.
   What is the probability of getting
   a. two 5s?
   b. two identical numbers?
   c. two different numbers?
   d. a total of 3 or less?
   e. a total of more than 3?

5. A box of chocolates contains hard centres, soft centres and liquid centres.
   $\frac{1}{2}$ of the chocolates have a hard centre and $\frac{3}{5}$ have a soft centre.
   Simon is first to choose a chocolate.
   What is the probability that he chooses a chocolate with a liquid centre?

6. Josef has two normal dice. He rolls the two dice and records the numbers shown on them, e.g. 2 and 5, 1 and 4 and 6 etc.
   a. How many possible outcomes are there?
   b. What is the probability of getting at least one prime number?
   c. What is the probability of getting a total between 8 and 12?
      (Do not include 8 and 12.)
   d. Are the outcomes in parts b and c mutually exclusive?

7. A box contains 150 biscuits. The probability of picking a shortbread biscuit is $\frac{1}{5}$, a chocolate-chip biscuit is $\frac{1}{10}$, a wafer biscuit is $\frac{1}{15}$ and a coconut biscuit is $\frac{1}{6}$.
   The rest of the biscuits in the box are custard creams.
   a. What is the probability of picking a custard cream?
   b. How many custard creams are in the box?
1. Write the ratio £3 : 20p in its simplest form.

2. Imagine flipping a coin twice.
   a. Draw a tree diagram to represent the possible outcomes.
   b. What is the probability of the coin landing heads both times?
   c. What is the probability of the coin landing heads once and tails once?

3. In a bag there are five 10p coins and five 2p coins. One coin is picked at random from the bag and is put back. Another coin is then picked at random from the bag.
   a. Draw a tree diagram to represent the possible outcomes.
   b. What is the probability of picking two 10p coins?
   c. What is the probability of picking one 10p and one 2p coin?

4. James has two boxes of sweets.
   Box A contains 10 toffees and 5 mints.
   Box B contains 10 toffees and 2 truffles.
   James selects one sweet at random from each box.
   a. Copy and complete this tree diagram.
   b. What is the probability that James selects two toffees?
   c. What is the probability that James selects no toffees at all?

5. In a bag there are five red balls and three blue balls. One ball is picked at random from the bag and is put back. Another ball is then picked at random from the bag.
   a. Draw a tree diagram to represent the possible outcomes.
   b. What is the probability of picking two red balls?
   c. What is the probability of picking one red and one blue ball?

6. Ceri has two boxes of chocolates.
   Box A contains 12 dark chocolates and 8 milk chocolates.
   Box B contains 8 dark chocolates, 10 milk chocolates and 2 white chocolates.
   Ceri selects one sweet at random from each box.
   a. Copy and complete this tree diagram.
   b. What is the probability that Ceri selects two dark chocolates?
   c. What is the probability that Ceri selects one milk and one white chocolate?

7. In a survey of 100 dinner ladies, 14 were vegetarians and the rest were not.
   Of the vegetarians, 2 wore glasses and 4 wore contact lenses.
   Of the non-vegetarians, 6 wore glasses and 25 wore contact lenses.
   Draw a tree diagram to represent this information, giving the probabilities on each branch.
9.4 Relative Frequency

1. Use equivalent fractions to write these fractions in ascending order.
   \( \frac{1}{5}, \frac{3}{5}, \frac{1}{2}, \frac{1}{20}, \frac{1}{4}, \frac{1}{10} \)

2. A supermarket survey of 100 shoppers found that eight of them only go to the supermarket on Sundays.
   In an average week, the supermarket has 20,000 customers.
   How many of these customers are likely to go to the supermarket only on Sundays?

3. A four-sided dice, numbered 1 to 4, is rolled.
   The result is recorded.
   The dice is then rolled again and the two results are added to give a total score.
   This is repeated to give 80 total scores.
   How many times in the 80 trials would you expect to get a total of
   a. 2
   b. 3?

4. In a school there are 900 pupils. There are 468 girls.
   a. What is the relative frequency of girls?
   b. In Year 7 there are 260 pupils.
      How many of the pupils in Year 7 are likely to be girls?

5. A Year 8 pupil did a survey on the shoe size of the 30 pupils in her maths class.
   The results are displayed in the frequency diagram.
   a. What is the estimated probability that a pupil in another Year 8 maths class will have a shoe size of 4?
   b. What is the relative frequency of a pupil having a shoe size of 6 or more?
   c. In the whole of Year 8 there are 300 pupils.
      How many pupils would you expect to have a shoe size of 7?

   a. Flip the coin 10 times and record how many times you get heads.
   b. Estimate the probability from these trials?
   c. Copy the relative frequency diagram and plot your results.
   d. Repeat the experiment another 40 times.
      Combine your results and record the probability (relative frequency) every 10 flips.
   e. What do you notice about your results?
10.1 Testing for congruence

1. Use a calculator to work out these.
   a. $4\frac{1}{2} + 3\frac{2}{3}
   b. 3\frac{1}{2} - 2\frac{3}{7}
   c. $8\frac{2}{5} \times \frac{6}{7}$

2. This diagram is made from three congruent isosceles triangles. Calculate the size of angle $a$.

3. This diagram is made from four congruent kites. Calculate the size of angle $b$.

4. Out of the three triangles described below, two are congruent and one is not. By constructing the triangles, or otherwise, find the odd triangle out.
   - Triangle $ABC$ with $AB = 6$ cm, $BC = 8$ cm and $CA = 10$ cm
   - Triangle $DEF$ with $DF = 10$ cm, $DE = 6$ cm and $\angle DFE = 30^\circ$
   - Triangle $GHI$ with $GI = 10$ cm, $HI = 8$ cm and $\angle HIG = 37^\circ$

5. Look at this diagram of two triangles, sharing the same base, drawn inside a circle.
   One triangle has the third vertex on the circumference of the circle, and the other has the third vertex on the centre of the circle. There is a mathematical theorem which states that angle $x$ is always twice the size of angle $y$.
   a. Draw three circles and check this theorem for yourself, using different size triangles.
   b. Have you created a practical demonstration or a proof?

6. $ABCD$ is a kite. Prove that triangle $ABD$ is congruent to triangle $CBD$.

7. $EFGH$ is a rectangle. Prove that triangle $EFH$ is congruent to triangle $GHF$. 

64 10.1 Testing for congruence
1. Use your calculator to work out $\sqrt{2345.408}$

2. a Describe fully a transformation that will map
   i. shape A onto shape B
   ii. shape A onto shape C
   iii. shape C onto shape D
   iv. shape B onto shape C.

   b Are shapes A, B, C and D congruent? Give a reason for your answer.

3. a Draw $x$- and $y$-axes from $-8$ to $8$.
   Draw the shape with vertices at $(2, 2), (7, 2), (5, 5)$ and $(3, 5)$. Label the shape A.

   b Rotate shape A $90^\circ$ anticlockwise about the point $(0, 1)$ and then reflect it in the line $y = 2$. Label your new image B.

   c Reflect shape B in the $y$-axis then translate it one square right and one square up. Label your new image C.

   d What single transformation will map shape C onto shape A?

4. a Describe fully a combination of a reflection and a translation that will map shape A onto shape B.

   b Describe fully a single transformation that will map shape B onto shape C.

   c Describe fully a single transformation that will map shape C onto shape A.

   d Explain why no combination of translations, reflections or rotations can map shape C onto shape D.

5. Shape K is a kite. The coordinates of its vertices are given in the table. Shape K undergoes a series of three transformations.
   a Copy and complete the table without drawing a diagram.

<table>
<thead>
<tr>
<th>Vertices</th>
<th>1st transformation in $y$-axis</th>
<th>2nd transformation in $x$-axis</th>
<th>3rd transformation Translate 5 right and 1 up</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 1)</td>
<td></td>
<td>$(-2, 1)$</td>
<td>$(-2, -1)$</td>
</tr>
<tr>
<td>(3, 3)</td>
<td></td>
<td></td>
<td>(3, 0)</td>
</tr>
<tr>
<td>(2, 4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1, 3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b What shape is the final image? How do you know?
1. a Which of the numbers on the cards below are cube numbers?

400 R  216 S  27 I  64 E  81 R  125 N
96 P  1 A  45 T  56 T  8 L  49 P

b Write down the letters from the cards with cube numbers on, starting with the smallest cube number. What word have you written?

Crop circles are patterns created by the flattening of crops such as wheat or barley. Many people believe that they are formed by UFOs landing in farmers' fields, but others believe that they are man-made. The name 'crop circle' was invented by a researcher, Colin Andrews, in the early 1980s, and was added to the Oxford English Dictionary in 1997.

As part of the crack UFO hunter team, you are going to investigate the crop circle pattern shown on the map below.

The scale of the map is 1 : 5000.

2. With a ruler, measure the diameter, $d$, of the largest circle in the pattern. What is the diameter of this circle in real life?

3. Measure and work out the real-life diameters of the smaller and smallest circles in the pattern.

4. What is the total real-life length of the crop circle pattern?

5. Work out the total area of the real-life crop circle pattern.

6. The total area of the field is 0.16 km$^2$. The crop flattened by the crop circle pattern has been ruined. What percentage of the total area of the field has been ruined?
10.4 Similarity

1. Write the next two terms of each of these sequences.
   a. 5, 6, 8, 11, ...
   b. 3, 6, 12, 24, ...
   c. 25, 36, 49, 64, ...
   d. 100, 96, 82, 70, ...

2. Write true or false for each of these.
   a. The mid-point of the line joining (5, 3) to (11, 3) is (8, 3).
   b. The mid-point of the line joining (-2, -4) to (-2, 6) is (-2, -1).
   c. The mid-point of the line joining (6, 5) to (6, -13) is (6, -8).
   d. The mid-point of the line joining (-1, 4) to (-7, 4) is (-4, 4).

3. The cards below show the coordinates of the points A to I. Put the cards into three groups of three. Each group should have the coordinates of the two end points of a line and its mid-point.

   A (-2, 5)   B (2, 0)   C (0, -3)   D (4, 0)   E (0, 3)
   F (0, 0)   G (-2, 1)   H (6, 0)   I (-2, 3)

4. The point M divides the line LN in the ratio 1:3. Work out the coordinates of M.

5. Triangle ABC and DEF are similar. Find the values of
   a. the length BC
   b. the length DE
   c. \( \angle ACB \).

6. Are the two triangles in this diagram similar? Give reasons for your answer.

7. Find the length of
   a. AB
   b. CD
10.5 Introducing trigonometry

1. In a sale, Glyn buys three CDs priced at £8.98 each and four DVDs priced at £12.99 each. The total price is reduced by 35% at the checkout.
   a. Write down one way of calculating the final price with a single calculation.
   b. Work out the final price using a calculator.

2. Using a calculator, write down the value of each ratio to 3 d.p.
   a. \( \sin 38^\circ \)  
   b. \( \tan 42^\circ \)  
   c. \( \cos 84^\circ \)

3. Calculate angle \( x \) in each of these.
   Give your answer to the nearest degree.
   a. \( \sin x = 0.9659 \)  
   b. \( \cos x = 0.9659 \)  
   c. \( \tan x = 0.9659 \)
   d. \( \cos x = 0.2231 \)  
   e. \( \tan x = 2.5 \)  
   f. \( \sin x = 0.6 \)

4. For each of these triangles, use the sine ratio to find the length of the side opposite to the given angle. Give your answers to 2 d.p.
   a. 
   b. 
   c. 

5. For each of these triangles, use the cosine ratio to find the length of the side adjacent to the given angle. Give your answers to 2 d.p.
   a. 
   b. 
   c. 

6. For each of these triangles, use the tangent ratio to find the length of the side opposite to the given angle. Give your answers to 2 d.p.
   a. 
   b. 
   c. 

7. For each of these triangles, use the appropriate trigonometric ratio to find the length of the side labelled \( x \). Give your answers to 2 d.p.
   a. 
   b. 
   c. 

Play any game on the LiveText CD.
1. Copy and complete.
   a) $4,320,000 = \underline{\quad}$ million
   b) $51,600 = \underline{\quad}$ thousand
   c) $\underline{\quad} = 3.5$ thousand
   d) $\underline{\quad} = 0.2$ million

2. For each of these triangles, use the sine ratio to find the length of the hypotenuse. Give your answers to 2 d.p.
   a) 5 cm
   b) 3 cm
   c) 5 cm

3. For each of these triangles, use the cosine ratio to find the length of the hypotenuse. Give your answers to 2 d.p.
   a) 4.7 cm
   b) 7.2 cm
   c) 8 cm

4. For each of these triangles, use the tangent ratio to find the length of the side adjacent to the given angle. Give your answers to 2 d.p.
   a) 20 cm
   b) 18 cm
   c) 15 cm

5. For each of these triangles, use the appropriate trigonometric ratio to find the length of the side labelled $x$. Give your answers to 2 d.p.
   a) 9.7 cm
   b) 6.8 cm
   c) 4.4 cm

6. James needs new tiles on the roof of his house. The roof has a height in the middle of 2 m and James knows that the angle of the roof is $35^\circ$. Calculate the length of the roof, marked $x$ in the diagram. Give your answer to the nearest centimetre.

7. During netball training the coach makes a square out of cones. The diagram shows the square $ABCD$. The length of the diagonal is 20 m. Players run from $A$ to $B$ to $D$ to $C$ to $A$. They do this 3 times. Calculate the total distance each player runs. Give your answer to the nearest metre.
11.1 Simplifying algebraic expressions

1. a Use the numbers from the cloud to copy and complete these sequences.
   i 3, 6, 11, __, 27, 38, __, 66, ...
   ii 9, 10, 12, __, 19, 24, 30, __, 45, ...
   iii 56, 53, __, 41, 32, 21, __, -7, __, -24, ...
   b Which number from the cloud haven't you used?
   c What is the square of this number?

2. Multiply out each bracket and collect like terms.
   a 4(5a + 6x) - 7a
   b 4(5b + 6) - 7
   c 4(5c - 6y) + 7c(6c + 5) + 4y

3. Copy and complete the table.

<table>
<thead>
<tr>
<th>Unfactorised</th>
<th>Fully factorised</th>
</tr>
</thead>
<tbody>
<tr>
<td>15x² + 10</td>
<td>4(x² + 2)</td>
</tr>
<tr>
<td>12x + 18b</td>
<td></td>
</tr>
</tbody>
</table>

4. Rajit has done his maths homework. It is written out below.
   His sister says that two of his answers are wrong, but will not tell him which two.
   Find and correct Rajit's two wrong answers.

   Factorise these expressions completely.
   a) 15xy + 9yz + 12y² = 3y(5x + 3z + 4y)
   b) 10a²b + 60ab + 300bd = 2b(5a²c + 30ad + 10d)
   c) pq + 3p²q + qr² = q(p + 3p² + r²)
   d) 2d²e + 2de² + 2 = 2de(d² + e + 1)

5. For these identities, find the values of a, b and c.
   a) 3(5x - 2y) - 5(x + y) = ax + by
   b) 3(2x + 3y - 4) - 2(x - 4y - 1) = ax + by + c

6. Factorise these expressions completely.
   a) 4a²b² + 8a³b
   b) 9a²d² + 9a²e²
   c) 28a²f²g² + 7a²f³g² + 14a³f^4g⁴

7. Expand each bracket and simplify each expression.
   a) 4a²(b² + b³) + 3b³(2a² - 3)
   b) x³(x² + y²) - x²(y² + x³) - x(y² - x²)
   c) 3x³(a + a² - a³) - 2a³(4 - a + 2a²)
11.2 Working with double brackets

1. Copy and complete this prime factor decomposition of 90.
   \[90 = 2 \times 3 \times 3 \times \square\]
   \[= 2 \times 3 \times \square \times \square\]

2. a. Write down the side lengths of the yellow square.
   b. Write down the area of the yellow square using brackets.
   c. Write down the area of the yellow square without using brackets.

3. a. Write down the side lengths of the green rectangle.
   b. Write down the area of the green rectangle using brackets.
   c. Write down the area of the green rectangle without using brackets.

4. Copy the table below. Match the question on the left to the working in the middle and the answer on the right. The first one is done for you.

<table>
<thead>
<tr>
<th>Question</th>
<th>Working</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ((x + 2)(x + 3))</td>
<td>(x^2 + 3x - 2x - 6)</td>
<td>(x^2 - x - 6)</td>
</tr>
<tr>
<td>b. ((x + 2)(x - 3))</td>
<td>(x^2 + 3x + 2x + 6)</td>
<td>(x^2 + x - 6)</td>
</tr>
<tr>
<td>c. ((x - 2)(x + 3))</td>
<td>(x^2 - 3x - 2x + 6)</td>
<td>(x^2 + 5x + 6)</td>
</tr>
<tr>
<td>d. ((x - 2)(x - 3))</td>
<td>(x^2 - 3x + 2x - 6)</td>
<td>(x^2 - 5x + 6)</td>
</tr>
</tbody>
</table>

5. Multiply out the brackets for each expression and then simplify.
   a. \((x + 5)(x + 5)\)
   b. \((2x + 5)(2x + 5)\)
   c. \((3x - 5)^2\)
   d. \((x + 5)(x - 5)\)
   e. \((2x - 5)(2x + 5)\)
   f. \((3x - 2)(3x + 2)\)

6. a. Multiply out \((x + y)(x - y)\).
   b. Use your result from part a to help you work out the value of \((x + y)(x - y)\) when \(x = 30\) and \(y = 12\).

7. Multiply out the brackets in this expression and then simplify.
   \[
   \frac{(2x + 2y)^2 - (2x - 2y)^2}{4}
   \]
1. Here are the nutrition information panels from two packets of cheese.

<table>
<thead>
<tr>
<th>Cheddar cheese</th>
<th>Somerset Brie</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>protein</strong></td>
<td><strong>protein</strong></td>
</tr>
<tr>
<td>25%</td>
<td>73.2 g</td>
</tr>
<tr>
<td><strong>carbohydrate</strong></td>
<td><strong>carbohydrate</strong></td>
</tr>
<tr>
<td>0.1%</td>
<td>1.6 g</td>
</tr>
<tr>
<td><strong>fat</strong></td>
<td><strong>fat</strong></td>
</tr>
<tr>
<td>34.4%</td>
<td>90 g</td>
</tr>
<tr>
<td><strong>fibre</strong></td>
<td><strong>fibre</strong></td>
</tr>
<tr>
<td>0%</td>
<td>0 g</td>
</tr>
<tr>
<td><strong>sodium</strong></td>
<td><strong>sodium</strong></td>
</tr>
<tr>
<td>0.7%</td>
<td>2 g</td>
</tr>
</tbody>
</table>

a) Write, as a percentage, the proportion in the Somerset Brie of
   i) protein  
   ii) carbohydrate.

b) Which cheese has the higher proportion of sodium?

c) Which cheese would be better for a low fat diet?

2. a) Write down the length and width of the blue rectangle.
   b) Write down the area of the rectangle using brackets.
   c) Write down the area of the rectangle without using brackets.

3. Copy and complete these equations.
   a) $x^2 + 8x = x(x + □)$
   b) $x^2 + 10x + 9 = (x + □)(x + □)$
   c) $x^2 + 9x + 20 = (x + □)(x + □)$

4. Factorise each of these expressions.
   a) $x^2 + 5x + 6$
   b) $x^2 + 11x + 30$
   c) $x^2 + 15x + 56$

5. Factorise these expressions into the form $(x + a)^2$.
   a) $x^2 + 8x + 16$
   b) $x^2 + 12x + 36$
   c) $x^2 + 16x + 64$

6. Which is the correct factorisation of each of these, A, B or C?
   a) $x^2 - 4$
      A) $(x + 2)(x - 2)$
      B) $(x - 2)^2$
      C) $(x - 2)(x - 2)$
   b) $x^2 - 100$
      A) $(x - 10)(x + 10)$
      B) $(x - 10)(x + 10)$
      C) $(x - 10)^2$

7. This fraction is made from two different quadratic expressions.

\[
\frac{x^2 + 5x}{x^2 + 7x + 10}
\]

a) Factorise the top and bottom of the fraction separately.
   b) Simplify by dividing the top and bottom by the common factor.
11.4 Using and writing formulae

1. Find the HCF of 396 and 420.

2. A rectangle has a width of \( x \) cm. Its length is twice its width.
   a. The width is 10 cm. Work out the area.
   b. Write an expression for the length of the rectangle.
   c. Write an expression for the area of the rectangle.
   d. Find the area when the length is 10 cm.

3. A personal fitness trainer charges her customers £\( p \) per hour for gym work and £\( r \) per hour for equipment work. She also charges £\( s \) per mile to travel to her customers. Write a formula for the cost, \( C \), for training a customer who lives \( m \) miles away for \( x \) hours of gym work and \( y \) hours of equipment work.

4. The surface area, \( S \), of a sphere is given by the formula \( S = 4\pi r^2 \), where \( r \) is the radius. Find the surface area of a sphere with radius 10 cm. Give your answer in terms of \( \pi \).

5. The surface area, \( S \), of this shape is given by the formula \( S = 2\pi r(2r + l) \).
   a. Work out the surface area when \( r = 5 \) cm and \( l = 10 \) cm.
   b. Work out the value of \( l \) when \( S = 100 \) cm\(^2\) and \( r = 2 \) cm.

6. Copy and complete the table to show the formulae for the area and perimeter of each of the shapes. All the formulae can be found in the cloud. The first one is done for you.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>half a circle</td>
<td>( \frac{1}{2}\pi r^2 )</td>
<td>( \pi r + 2r )</td>
</tr>
<tr>
<td>triangle</td>
<td>( \frac{1}{2}\pi r^2 )</td>
<td>( \pi r + 2r )</td>
</tr>
<tr>
<td>sector</td>
<td>( \frac{3}{4}\pi r^2 )</td>
<td>( \frac{3}{4}\pi r + 2r )</td>
</tr>
<tr>
<td>kite</td>
<td>( \frac{1}{2}\pi r + 2r )</td>
<td>( \pi r )</td>
</tr>
</tbody>
</table>

7. The volume, \( V \), of a square-based pyramid is given by the formula \( V = \frac{1}{3}S^2h \), where \( S \) is the side length of the base and \( h \) is the height.
   a. Find the volume of a square-based pyramid when \( S = 10 \) cm and \( h = 21 \) cm.
   b. Find the side length of a square-based pyramid when \( V = 1440 \) cm\(^3\) and \( h = 30 \) cm.
1. a) Copy these numbers. They are a coded message!

| 2 | 10 | 9 | 7 | 8 | 6 | 81 | 25 | 64 | 7 | 5 | 6 | 8 | 4 | ? |

b) Each number in the code is the value of one of the expressions in the key.
Replace each number with the matching letter to crack the code.
The first one has been done for you.

\[3^2 = 9\]
So 9 represents \(A\)
What is the secret message?

2. Mosal has rearranged these two equations.

\[4x + 3 = y\]
\[4x - 3 = y\]

He has made a puzzle by writing his workings on these four cards.

\[4x = y + 3\]
\[x = \frac{y - 3}{4}\]
\[4x = y - 3\]
\[x = \frac{y + 3}{4}\]

Solve the puzzle by putting the correct workings with the correct equation.

3. Solve the equation \[6x + 4 = 3x + 25\]

4. Write each of these in the form \(y = mx + c\).
   a) \(2y = 8x + 4\)
   b) \(3y - 6x = 12\)
   c) \(3y + 12x = y - 6\)

5. Look at this diagram of a square inside a circle.
   Write a formula for the red area, \(A\), in terms of the side length of the square \(a\), and the radius of the circle, \(r\).

6. The surface area, \(S\), of a sphere is given by the formula \(S = 4\pi r^2\).
   Make \(r\) the subject of the formula.

7. This formula has \(x\) on both sides.
   \(px + 3f = 8f - qx\)
   Copy and complete this rearrangement to make \(x\) the subject of the formula.
   \(px + \underline{\quad} = 8f - 3f\)
   \(x(p + \underline{\quad}) = 5f\)
   \(x = \frac{5f}{(p + \underline{\quad})}\)

8. Make \(x\) the subject of each of these formulae.
   a) \(5x + 2a = 8p\)
   b) \((x + y)^2 = z\)
   c) \(\frac{2x}{5} + y = a\)
11.6 Inequalities

1. In this number wheel, opposite numbers multiply together to give -24. Copy the wheel and fill in the missing numbers.

2. Look at this number line.

\[\begin{array}{cccccc}
& -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}\]

a. Write down the inequality shown by the red line.
b. Write down the inequality shown by the blue line.

3. Write down all the integer solutions to each of these inequalities.
   a. \(3.5 \leq x \leq 6\)
   b. \(-3.5 < x \leq 3.5\)
   c. \(-3.5 < 2x \leq 3.5\)

4. a. Draw the line \(y = 2x + 2\).
   b. Shade the region \(y \leq 2x + 2\).

5. Look at this diagram.

\[\begin{array}{c}
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
-2 & -1 & 0 & 1 & 2 & 3 & 4 \\
\end{array}\]

a. Write down the equation of each coloured line.
b. Write down the three inequalities that are satisfied by the yellow region.

6. Show on a graph the region enclosed by these inequalities.
   a. \(y > 4x\) and \(-3x < 12\)
   b. \(5 < 5x - 5 < 15\) and \(x + y < 4\)

7. a. Sketch \(y = x^2\) and \(y = 8\) on the same graph.
   b. Shade the area of your graph that satisfies \(y > x^2\) and \(y < 8\).
   c. Show the values of \(x\) within the shaded region on a number line and express this as an inequality using integer values.
12.1 Data problem solving

1. Use a written method to work out $2437 + 34.82 + 0.96 - 187.7$

2. Plan an investigation that has something to do with cars. Clearly state the aims of your investigation.

3. a. State what data you will need to collect to test the hypothesis of the investigation you planned in Q2.
   b. How much data will you need, to make your conclusions valid?
   c. How will you record the outcomes of your data collection?
   d. How will you avoid bias?

4. Morgan collected information about the time taken for his classmates to complete their geography homework. His results are summarised in the table.
   
<table>
<thead>
<tr>
<th>Time (min)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>5</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

   a. Use the data to draw an appropriate graph.
   b. Use the data to carry out an appropriate calculation.

5. An estate agent is looking at the sale prices of three-bedroom houses sold in the last year. He also looks at the distances of the houses from the nearest train station. Here are his results.

<table>
<thead>
<tr>
<th>Distance from nearest station (miles)</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sale price of house (£ thousands)</td>
<td>240</td>
<td>210</td>
<td>190</td>
<td>230</td>
<td>205</td>
<td>195</td>
<td>225</td>
<td>235</td>
<td>209</td>
<td>188</td>
<td>212</td>
<td>220</td>
</tr>
</tbody>
</table>

   a. Draw a graph to show this data.
   b. Describe the trend that your graph in part a shows.
   c. Estimate the cost of a three-bedroom house that is four miles from the nearest train station.

6. Polly is designing a new game. She has four identical counters. One side of each counter says ‘Truth’, the other side says ‘Dare’. She flips all four counters at the same time.

   a. Draw a tree diagram to show all the possible outcomes.
   b. How many different outcomes are there?
   c. What is the probability of getting all four counters saying ‘Dare’?

7. Look again at the information in Q4.
   Morgan says:
   
   I can now tell how long, on average, pupils in my school take to do their geography homework.

   Explain why Morgan is wrong. How could he make his findings more reliable?
1. Work out the area of this parallelogram. Give your answer to the nearest square centimetre.

2. ‘Halving a number always makes the number smaller.’ Is this statement true? Justify your answer.

3. Paul buys a new car. He puts his old car up for sale for £400. If he doesn’t sell the car, at the start of each following week he will reduce the price by 10% and then round the new price to the nearest £10.
   a. At the start of the second week, what is the price of Paul’s old car?
   b. At the start of the fifth week, what is the price of Paul’s old car?
   c. Paul finally sells the car for £150. During which week did he sell it?

4. Three friends go out for a meal. Here is the menu.

<table>
<thead>
<tr>
<th>Starters</th>
<th>Main Courses</th>
<th>Puddings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garlic bread £3.75</td>
<td>Chicken risotto £9.50</td>
<td>Tiramisu £4.50</td>
</tr>
<tr>
<td>Garlic mushroom</td>
<td>Green risotto £8.35</td>
<td>Flan £4.25</td>
</tr>
<tr>
<td>Calamari £4.25</td>
<td>Pasta carbonara £7.45</td>
<td>Fudge cake £4.65</td>
</tr>
<tr>
<td>Olives £2.75</td>
<td>Meat feast pizza £9.00</td>
<td>Stuffed figs £4.20</td>
</tr>
<tr>
<td>Mixed salad £3.25</td>
<td>Four cheese pizza £8.75</td>
<td>Ice cream £3.50</td>
</tr>
</tbody>
</table>

   Special offers
   - Starters - buy 2 for £5
   - Main course - buy 2 and get the cheapest free

   The friends order garlic bread, calamari, olives, chicken risotto, pasta carbonara, four-cheese pizza, tiramisu, flan and ice cream.
   Their drinks cost £12.35. They leave a tip of 10% of the total cost of the meal and drinks. They share the total cost equally between them.
   a. Use the special offers to work out the cheapest total bill.
   b. Calculate how much they each pay.

5. The table shows the amounts of oil produced by the top five oil-producing countries in 2007. The total world oil production in 2007 was $8.554 \times 10^7$ barrels per day.
   a. Which country’s oil production isn’t written in standard form?
   b. Approximately how many barrels of oil did Russia produce in 2007?
   c. Approximately how many times bigger than China’s production was the total world oil production in 2007?
   d. What percentage of the total world oil production was produced by the top five countries together in 2007? Give your answer to the nearest 1%.

<table>
<thead>
<tr>
<th>Country</th>
<th>Oil production (barrels per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saudi Arabia</td>
<td>$1.025 \times 10^7$</td>
</tr>
<tr>
<td>Russia</td>
<td>$9.98 \times 10^6$</td>
</tr>
<tr>
<td>USA</td>
<td>$84.57 \times 10^5$</td>
</tr>
<tr>
<td>Iran</td>
<td>$4.7 \times 10^6$</td>
</tr>
<tr>
<td>China</td>
<td>$3.72 \times 10^6$</td>
</tr>
</tbody>
</table>
12.3 Algebra problem solving

1. I think of a number \( x \), add 5 and multiply the result by 8. The answer is 96. Form an equation and solve it to find the value of \( x \).

2. The sum of two consecutive odd numbers is 368. Use an algebraic method to find the value of these integers.

3. Each face of a tetrahedron has an area of \((5y - 6)\) cm\(^2\). The surface area of the tetrahedron is 146 cm\(^2\).
   a. Write an equation connecting the area of each face and the surface area of the tetrahedron.
   b. Solve the equation to find the value of \( y \).

4. A cuboid has two faces with an area of \( 4x^2 \) and four faces with an area of \( 8x^2 \).
   Three of the cuboids are used to make the shape A.
   a. Write an expression for the surface area of shape A.
   b. Write an expression for the length, height and width of one cuboid.
   c. Write an expression for the volume of shape A.
   d. The actual surface area of shape A is 3168 cm\(^2\).
      Write an equation using the expression you formed in part a and solve it to find \( x \).
   e. What are the length, height and width of one cuboid?
   f. What is the volume of shape A?

5. For each mathematical statement write down one example to show that it is not always true.
   a. \( x^3 > 3x \)
   b. \( x > \frac{1}{x} \)
   c. \(-2x < 2x\)

6. In a café the Wilson family spend £5.15 on two mugs of coffee and three mugs of tea.
   In the same café, the Patel family spend £8.20 on three mugs of coffee and five mugs of tea.
   Form a pair of simultaneous equations and solve them to find the price of one mug of coffee and one mug of tea.

7. On this number grid Greg draws the cross shown. He calls it the 13-cross, as 13 is the number at the centre.
   The total of the numbers in the 13-cross is \( 3 + 12 + 13 + 14 + 23 = 65 \)
   Greg says, ‘The total of any cross I draw on the grid will always be divisible by 5.’
   a. Show that this is true for the 28-cross.
   b. Greg decides to prove that his statement is true using algebra, so he draws the \( n \)-cross.
      Copy and complete the \( n \)-cross.
   c. Use algebra to prove that Greg’s statement is always true.
12.4 Geometrical reasoning: lines, angles and shapes

1. Which two of these shapes have the same volume?
   - A
     - 2 cm
     - 3 cm
     - 4 cm
   - B
     - 2 cm
     - 2 cm
     - 5 cm
   - C
     - 1 cm
     - 8 cm
     - 3 cm

2. Calculate the size of angle $x$.
   State the angle properties you are using at every stage of your working.

3. Calculate the size of angle $y$.
   State the angle properties you are using at every stage of your working.

4. This diagram shows the floor plan of a room. The area of the room is 16.11 m$^2$.
   Adam puts skirting board along the sides of the room shown by the red line.
   Work out the total length of skirting board that Adam needs.

5. A hockey pitch has a length of 91.4 m.
   The length of a diagonal is 106.7 m.
   Work out the area of the hockey pitch.
   Give your answer to the nearest m$^2$.

6. Triangle $ABC$ is isosceles.
   The base length $BC = 12$ cm, and $\angle CAB = 42^\circ$.
   Use this information to find
   a. the perimeter of the triangle
   b. the area of the triangle.
   Justify each of your answers.

7. The diagram shows rectangle $ABCD$.
   $AB = 5x$ cm and $BD = 10x$ cm.
   a. Show that $AD = 5\sqrt{3}x$ cm.
   b. Find the area of the rectangle in terms of $x$,
      leaving your answer in surd form.
12.5 Percentage and proportion problems

1. Use a written method to work out $2.45 \times 3.7$

2. Which offer is the better value? Give a full and clear answer.
   - 25% off! Six-pack of ready salted crisps. Normal price: £2.52
   - Buy a 12-pack of ready salted crisps for the price of nine! Nine-pack price: £4.05

3. Which of these graphs show variables that are always in direct proportion?

   A  B  C  D

4. Write true or false for each of these.
   a. The time taken to wash windows and the number of windows are two variables that are always in direct proportion.
   b. The total cost of some milk and the number of litres bought are two variables that are always in direct proportion.
   c. The cost of a mobile phone and the cost of sending text messages are never in proportion.

5. Tyra goes to Switzerland for a business meeting. When she goes, the exchange rate is £1 = 1.6 Swiss francs. The number of pounds $p$ is directly proportional to the number of Swiss francs $f$.
   a. Write down a formula connecting the number of pounds $p$ to the number of Swiss francs $f$.
   b. How many Swiss francs would Tyra get in exchange for £80?
   c. If Tyra wanted 180 Swiss francs, how many pounds would she need to exchange?
   d. Use a graph to represent the information. How can you tell that the values are in direct proportion?

6. Sian, Shaun and Shona all decide to buy a new car. The price of the car is £8000. There are three different payment plans available.

   **Plan 1**
   - Deposit: £2000
   - Interest of 5% charged on the remainder, then payments spread equally over 12 months

   **Plan 2**
   - Deposit: £3000
   - Nine equal monthly payments of £580

   **Plan 3**
   - Deposit: None
   - Interest of 8% charged on the full price, then payments spread equally over 18 months

   Sian has savings of £2200 and can afford to pay £500 per month.
   Shaun has savings of £3000 and can afford to pay £600 per month.
   Shona has savings of £3500 and can afford to pay £550 per month.

   Which of the payment plans would be suitable for each person? Justify your answers with appropriate calculations and explanations.
12.6 Functions and graphs

1. Use the brackets or memory keys on your calculator to work out these.
   Give your answers to two decimal places.
   a. \[14.3 - 2.87 \div 3.35 + 0.88\]
   b. \[(2.4 + 9.55)^2 \div 52.3 \times 0.22^3\]
   c. \[\sqrt{90} + 3.67 \div 48 \times 1.5^2\]

2. a. Copy and complete the table.
   \[\begin{array}{c|c|c|c|c|c}
   x & -3 & -1 & 0 & 2 & 3 \\
   \hline
   y = 3x + 1 & & & & & \\
   \end{array}\]

   b. Write the values in the table as coordinate pairs.

3. a. Use Q2 to help you plot the graph of \(y = 3x + 1\).
   b. Identify two other points you could have used to plot the same graph.
   c. On the same axes draw the graph of \(y = 3x - 2\).
   d. What do you notice about your graphs?

4. a. Use the graph to copy and complete the table.
   \[\begin{array}{c|c|c|c|c}
   x & -4 & -2 & 0 & 2 & 4 \\
   \hline
   y & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\
   \frac{1}{2}x & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\
   \end{array}\]

   b. Copy and complete:
   The equation of the line is \(y = \frac{1}{2}x = \ldots\).
   c. Rearrange the equation of the line so that it begins \(y = \ldots\).

5. Match the cards into pairs of equations that represent the same line.

   - \(y - x = 4\)
   - \(y = 4 - 2x\)
   - \(y + 2x = 4\)
   - \(y = x + 4\)
   - \(y + 4 = 2x\)
   - \(y + x = 4\)
   - \(y = 4 - x\)
   - \(y = 2x - 4\)

6. Tom needs an electrician to do some work in his house that should take about 3 hours. He needs to decide between these two electricians.
   a. Represent this information graphically by working out each electrician's price for work lasting from one to eight hours.
      Use your graph to make a recommendation for who Tom should use.
   b. Represent the information algebraically by writing a pair of simultaneous equations and solving them.
   c. When should customers use Electric Eddie and when should they use Laura Leccy if they want to get the cheapest price?
13.1 Questionnaires and samples

1. Match each yellow card with the correct blue card.

2. Look at the following data and answer the questions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cattle and calves</td>
<td>1359.9</td>
<td>134.0</td>
<td>127.0</td>
<td>119.5</td>
<td>128.1</td>
</tr>
<tr>
<td>Dairy breeding herd</td>
<td>256.7</td>
<td>276.3</td>
<td>268.6</td>
<td>267.7</td>
<td>271.5</td>
</tr>
<tr>
<td>Beef breeding herd</td>
<td>219.3</td>
<td>225.3</td>
<td>224.4</td>
<td>195.8</td>
<td>216.8</td>
</tr>
<tr>
<td>Total sheep and lambs</td>
<td>10,874.0</td>
<td>11,505.2</td>
<td>11,912.2</td>
<td>10,050.1</td>
<td>9,736.8</td>
</tr>
<tr>
<td>Total sheep, one year old and over</td>
<td>5893.1</td>
<td>5466.3</td>
<td>5872.5</td>
<td>5386.1</td>
<td>5221.1</td>
</tr>
<tr>
<td>Lambs under one year old</td>
<td>4980.4</td>
<td>5538.9</td>
<td>5319.7</td>
<td>4664.0</td>
<td>4524.8</td>
</tr>
<tr>
<td>Total pigs</td>
<td>98.8</td>
<td>93.0</td>
<td>68.1</td>
<td>44.3</td>
<td>30.7</td>
</tr>
<tr>
<td>Sows for breeding</td>
<td>1.2</td>
<td>1.6</td>
<td>0.8</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>Sows for fattening</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

a. What percentage of the ‘Total pigs’ were ‘Sows for breeding’ in 2004?
b. What percentage of the ‘Total sheep and lambs’ were ‘Lambs under one year old’ in 2000?
c. Which out of ‘Total cattle and calves’, ‘Total sheep and lambs’ and ‘Total pigs’ had the largest percentage decrease from 1996 to 2004?
d. During which two years was there the biggest percentage decrease in the total number of pigs?

3. You are conducting a survey about the types of holidays that people like to go on. Where should you position yourself in order to carry out your survey? Explain your reasons for choosing or not choosing each one of these locations.
   a. outside a camping shop
   b. at the nearest airport
   c. outside a supermarket
   d. at the ski show
   e. outside a cinema
   f. outside the town hall

4. Decide whether each of these questions is biased or not. If it is, re-write it so that it is suitable for a questionnaire.
   a. Fast food is bad for you. Do you agree?
      Yes   No   Don’t know
   b. What time do you get up on a weekday morning?
      Before 7 am   Between 7 am and 8 am   After 8 am
   c. What type of milk do you buy?
      Skimmed   Semi-skimmed
   d. Do you agree that girls are better than boys at maths?
      Yes   No   Don’t know
13.2 Averages from grouped data

1. Which of the following points lie on the line \( y = 2x + 5 \)?
   - A (1, 4)
   - B (3, 11)
   - C (5, 14)
   - D (0, 7)
   - E (−1, 3)

2. The table shows the lifespans of high intensity projector bulbs. Calculate an estimate of the mean lifespan of the bulbs.

<table>
<thead>
<tr>
<th>Lifespan (hours)</th>
<th>Frequency</th>
<th>Mid-point</th>
<th>Frequency × mid-point</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 ≤ hours &lt; 200</td>
<td>7</td>
<td>150</td>
<td>1050</td>
</tr>
<tr>
<td>200 ≤ hours &lt; 300</td>
<td>18</td>
<td>250</td>
<td>4500</td>
</tr>
<tr>
<td>300 ≤ hours &lt; 400</td>
<td>31</td>
<td>350</td>
<td>10850</td>
</tr>
<tr>
<td>400 ≤ hours &lt; 500</td>
<td>42</td>
<td>450</td>
<td>18900</td>
</tr>
<tr>
<td>500 ≤ hours &lt; 600</td>
<td>2</td>
<td>550</td>
<td>1100</td>
</tr>
</tbody>
</table>

3. The table shows the weights, to the nearest gram, of 100 puffin chicks. Calculate an estimate of the mean weight of the puffin chicks.

<table>
<thead>
<tr>
<th>Weight, ( w ) (grams)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 ≤ ( w ) &lt; 350</td>
<td>2</td>
</tr>
<tr>
<td>350 ≤ ( w ) &lt; 400</td>
<td>7</td>
</tr>
<tr>
<td>400 ≤ ( w ) &lt; 450</td>
<td>72</td>
</tr>
<tr>
<td>450 ≤ ( w ) &lt; 500</td>
<td>18</td>
</tr>
<tr>
<td>500 ≤ ( w ) &lt; 550</td>
<td>1</td>
</tr>
</tbody>
</table>

4. This table shows the ages of the people at a cricket club one Sunday. Calculate an estimate for the median age of the people at the cricket club.

<table>
<thead>
<tr>
<th>Age, ( y ) (years)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ≤ ( y ) &lt; 20</td>
<td>22</td>
</tr>
<tr>
<td>20 ≤ ( y ) &lt; 30</td>
<td>30</td>
</tr>
<tr>
<td>30 ≤ ( y ) &lt; 40</td>
<td>12</td>
</tr>
<tr>
<td>40 ≤ ( y ) &lt; 50</td>
<td>10</td>
</tr>
<tr>
<td>50 ≤ ( y ) &lt; 60</td>
<td>4</td>
</tr>
<tr>
<td>60 ≤ ( y ) &lt; 70</td>
<td>13</td>
</tr>
</tbody>
</table>

5. This frequency polygon shows the numbers of goals scored by a school football team last season. Use the frequency polygon to calculate the mean number of goals scored per game last season.

6. Use the frequency polygon in Q5 to find the median number of goals scored last season.
1. Copy and complete this number pyramid. The number in each brick is the sum of the numbers in the two bricks below it.

\[ -19 + 12 = -7 \]

2. In one school, a total of 95 pupils had a competition to see whether Year 8 or Year 9 pupils were better at Cliff Hanger. Here are the partially completed cumulative frequency tables of the Year 8 and Year 9 results.

**Year 8 results**

<table>
<thead>
<tr>
<th>Distance, ( d ) (cm)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 &lt; d \leq 5.0)</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(5.0 &lt; d \leq 10.0)</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>(10.0 &lt; d \leq 15.0)</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>(15.0 &lt; d \leq 20.0)</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>(20.0 &lt; d \leq 25.0)</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>(25.0 &lt; d \leq 30.0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Year 9 results**

<table>
<thead>
<tr>
<th>Distance, ( d ) (cm)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 &lt; d \leq 5.0)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(5.0 &lt; d \leq 10.0)</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>(10.0 &lt; d \leq 15.0)</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>(15.0 &lt; d \leq 20.0)</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>(20.0 &lt; d \leq 25.0)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(25.0 &lt; d \leq 30.0)</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

a. Copy and complete the cumulative frequency tables.
b. Draw cumulative frequency diagrams for the Year 8 and Year 9 results on separate grids.
c. Use your cumulative frequency diagrams to work out the median and the upper and lower quartiles for each year group.
d. Calculate the interquartile range for each year group.
e. Compare the scores for both year groups. Who performed better? Use your results to justify your conclusions.
13.4 Scatter graphs and correlation

1. Steffi buys a new motorbike. She puts her old bike up for sale for £380. If she doesn’t sell the bike, at the start of each following week she will reduce the price by 15% and round the new price to the nearest £10.
   a. At the start of the second week, what is the price of Steffi’s old bike?
   b. At the start of the third week, what is the price of Steffi’s old bike?
   c. Steffi finally sells the bike for £170. During which week did she sell it?

2. Which type of correlation does each of these newspaper headlines describe?
   a. The less sleep you have, the shorter your attention span
   b. The less sleep you have, the bigger the bags under your eyes!

3. Match each of these scatter graphs to the correct type of correlation.
   a. [Scatter graph with upward trend]
   b. [Scatter graph with no correlation]
   c. [Scatter graph with downward trend]
   A. no correlation
   B. strong positive correlation
   C. very strong positive correlation

4. The table shows the percentage marks that the boys from 8C got in their maths and science tests.

<table>
<thead>
<tr>
<th>Pupil</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maths mark (%)</td>
<td>36</td>
<td>73</td>
<td>30</td>
<td>32</td>
<td>47</td>
<td>93</td>
<td>81</td>
<td>58</td>
<td>75</td>
<td>66</td>
<td>87</td>
<td>31</td>
<td>60</td>
<td>47</td>
</tr>
<tr>
<td>Science mark (%)</td>
<td>45</td>
<td>81</td>
<td>63</td>
<td>39</td>
<td>57</td>
<td>94</td>
<td>32</td>
<td>62</td>
<td>82</td>
<td>72</td>
<td>90</td>
<td>32</td>
<td>71</td>
<td>57</td>
</tr>
</tbody>
</table>

   a. Draw a pair of axes, with the vertical axis going from 30 to 100 and labelled ‘Science mark (%)’, and the horizontal axis going from 30 to 100 and labelled ‘Maths mark (%)’.
   b. Plot the data from the table to give a scatter graph.
   c. Describe the connection between the maths marks and the science marks.
   d. Use a transparent ruler to draw a line of best fit on the scatter graph.
   e. Pupil G was ill when he took the science test. Use your line of best fit to estimate what he probably would have scored in his science test if he had been well.
   f. Identify the other pupil who sat one of the tests when they were ill, and so does not follow the trend.
   g. Use your line of best fit to estimate what the pupil from part f should have scored.
13.5 Misleading graphs and charts

1. The sum of two consecutive multiples of 6 is 222. Use an algebraic method to find the value of these integers.

2. The table shows data on the favourite sports of the Year 8 girls in one school. Which type of graph or chart would make it easiest for the PE teachers to see what fraction of the girls liked the different sports?

<table>
<thead>
<tr>
<th>Sport</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>swimming</td>
<td>12</td>
</tr>
<tr>
<td>dance</td>
<td>27</td>
</tr>
<tr>
<td>netball</td>
<td>83</td>
</tr>
<tr>
<td>hockey</td>
<td>30</td>
</tr>
<tr>
<td>tennis</td>
<td>34</td>
</tr>
</tbody>
</table>

   - Bar chart
   - Pie chart
   - Line graph

3. This bar chart shows information about the numbers of merits given to the boys and girls of 8T in the last four weeks. Sam says, ‘The girls got twice as many merits in week 4 as they did in week 1.’ Explain why Sam is wrong.

4. This bar chart shows information about the numbers of merits given to the boys and girls of 8Y in the last four days.
   a. What is wrong with this bar chart?
   b. How could you improve it?

5. Miss Read gave her English class a simple crossword to do. She recorded how long it took each pupil to finish. The results are shown in the bar chart. Use the bar chart to say whether each of these statements is true or false, or whether there is not enough information to tell. Explain your answers.
   a. The median time was between 5 and 6 minutes.
   b. The fastest person took 3 minutes.
   c. No-one took more than 8 minutes.
   d. The girls got more correct answers than the boys.
   e. The boys finished, on average, faster than the girls.
13.6 Comparing distributions

1. Copy and complete this prime factor decomposition of 252.

\[ 252 = 2 \times 2 \times 3 \times \square \times \square \]

\[ = 2^2 \times 3 \times \square \times \square \]

2. John and Alistair play each other at golf. This table shows their mean numbers of shots played per hole and their ranges in the number of shots played per hole.

<table>
<thead>
<tr>
<th>Mean number of shots played per hole</th>
<th>Range in the number of shots played per hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>John 5.6</td>
<td>6</td>
</tr>
<tr>
<td>Alistair 7</td>
<td>2</td>
</tr>
</tbody>
</table>

a) Copy and complete these sentences.
   i) _____ is a better golfer on average because he has a lower _____.
   ii) _____ is a more consistent golfer because he has a lower _____.

b) On a ‘good’ day, who do you think gets the lower overall score?
c) On a ‘bad’ day, who do you think gets the higher overall score?
d) If you had to choose who was the better golfer, who would you choose? Explain your answer.

3. This frequency polygon shows the amounts of money spent by people in a village shop one day. Look at the shape of the distributions. Write three sentences to compare the amounts spent in the shop between 7 am and 8 am and the amounts spent between 1 pm and 2 pm.

4. These pie charts show the proportions of the different types of litter found on the beaches in Northern Ireland, Wales, Scotland and England.

Write a report describing what the charts tell you.
14.1 Solving geometrical problems

1. Find the highest common factor (HCF) of 24, 40 and 72.

2. Sam has drawn three triangles, measured their angles and written the sizes of the angles onto cards. She has lost one of the cards! These are the eight cards she has left.

   125°, 70°, 155°, 25°, 15°, 45°, 30°, 65°

   a. Work out which cards go together to make the triangles.
   b. What size is the angle on the card she has lost?

3. Carlos has some of these triangular tiles.
   He starts to fit them around a point, but they don't fit exactly.
   a. How many whole tiles can he fit around the point?
   b. How many degrees is the gap that is left?

4. Miguel has some of these tiles in the shape of a kite.
   He starts to fit them around a point using the angle at A, but they don't fit exactly.
   a. How many whole tiles can he fit around the point?
   b. How many degrees is the gap that is left?
   c. If he tried to fit them around a point using the angle at C would they fit exactly? Explain your reasoning.

5. A flagpole is 12 m high.
   A supporting wire is fixed from the ground to halfway up the flagpole as shown in the diagram.
   Work out the length of wire needed.
   Give your answer to the nearest cm.

6. Niall needs new tiles on both sides of his roof.
   The sketch shows the cross-section of his roof.
   a. What is the length of the roof, marked x on the sketch?
   b. The roof of Niall's house is 9.2 m wide.
      What is the total area, in m², of tiles that Niall needs for his roof?

7. Jorgen is building some decking in his garden.
   He wants to check that he has the length and the width at right angles. He measures the length, the width and the diagonal, as shown in the diagram.
   Is angle x a right angle? Explain your reasoning.
1. Isobel has spilt cola on her homework!
Work out the numbers that have been hidden by the cola stain.

- Find the prime factor decomposition of each of these numbers.
  Write your answers as powers.

  - $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
  - $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$
  - $96 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
  - $45 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$

2. Find the size of each lettered angle.
Give reasons for your answers.

3. Find the size of each lettered angle.
Give reasons for your answers.

4. Find the size of each lettered angle.
Give reasons for your answers.

5. This arrowhead is made from two identical scalene triangles.
Work out the size of angle $y$.
Justify your answer.

6. Alex has some of these isosceles trapeziums.
He puts them together as shown in the diagram.
How many more are needed to make a complete ring?
Explain your answer.

7. Stephanie makes a ring out of isosceles triangles and trapeziums. She puts them together by alternating them as shown in the diagram.
How many triangles and trapeziums are needed to make a complete ring?
Explain your answer.
1 Simplify each expression. Leave your answer as a power.
   a) $4^5 \times 4^7$
   b) $8^3 \times 8^9$
   c) $5^6 \div 5^2$
   d) $12^0 \div 12^6$
   e) $3^2 \times 3^5 \div 3^3$
   f) $q^8 \div q^2 \times q^3$

2 Match each of these solids to the correct plan view.

SOLIDS

PLAN VIEWS

A   B   C   D

front  side

3 Draw the front and side elevations for each of the solids in Q2.

4 This is a plan of a BMX circuit.
   It consists of straight sections and
   semicircular banks.
   Draw an accurate scale drawing
   using a scale of 1 : 500.

5 This is a scale drawing of Ian’s garden.
   The width of the garden is 16 m.
   a) By measuring the width, work out what 1 cm
      on the scale drawing represents in real life.
   b) What is the real-life length of the garden?

6 How many planes of symmetry does each of these solids have?
   a) 
   b) 
   c) Rectangular cuboid
   d) 

6c

6c

6b

7c

7c
1. This is Mair’s fractions homework. She has only got one question right.

<table>
<thead>
<tr>
<th>Question</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = \frac{5}{4})</td>
<td></td>
</tr>
<tr>
<td>2. (1\frac{3}{4} + 3\frac{4}{5} = 4 + \frac{3}{4} + \frac{4}{5} = 4 + \frac{15}{20} + \frac{16}{20} = \frac{25}{4})</td>
<td></td>
</tr>
<tr>
<td>3. (1\frac{1}{5} - \frac{1}{5} = \frac{6}{5} - \frac{1}{5} = \frac{5}{5} = 1)</td>
<td></td>
</tr>
<tr>
<td>4. (\frac{4}{3} - \frac{7}{36} = 2 + \frac{7}{36} - \frac{7}{36} = \frac{2}{36} = \frac{1}{18})</td>
<td></td>
</tr>
</tbody>
</table>

a. Check her homework. Which question has she got right?

b. Explain what she has done wrong in the other questions, and write out the correct solutions for her.

Where necessary, use the \(\pi\) button on your calculator.

2. Put the following prisms in order of size, starting with the smallest volume.

3. Put the prisms from Q2 in order of size, starting with the smallest surface area.

4. Work out the surface area of this curtain pole. Give your answer to two decimal places.

5. Calculate the volume of this gas storage cylinder. Give your answer to the nearest whole cubic metre.

6. The volume of this cylinder is \(99\ cm^3\). Calculate the height of the cylinder. Give your answer to the nearest mm.

7. The volume of this cylinder is \(1\ m^3\). Calculate the radius of the cylinder. Give your answer to the nearest cm.
1. Look at this sequence of shapes made from dots.

   - Shape 1: 6 dots
   - Shape 2: 10 dots
   - Shape 3: 14 dots

   a. Find the next two terms of the sequence.
   b. Find the $n$th term of the sequence.

2. Calculate the lettered lengths in these triangles.
   - a. \(a\) \(40^\circ\) \(13\ m\)
   - b. \(b\) \(70^\circ\) \(20\ m\)
   - c. \(c\) \(60^\circ\) \(18\ m\)

3. Calculate the lettered lengths in these triangles.
   Give your answers to three significant figures.
   - a. \(a\) \(15\ mm\)
   - b. \(b\) \(20\ m\)
   - c. \(c\) \(10\ m\)

4. Calculate the lettered angles in these triangles.
   Give your answers to one decimal place.
   - a. \(a\) \(15\ cm\)
   - b. \(b\) \(0.22\ m\)
   - c. \(c\) \(0.7\ m\)

5. Nelson's Column in London is 51.659 m tall. An architect stands 20 m from the base of the column and looks up to the top. What is the angle of elevation, shown as \(x\) in the diagram?
14.6 Solving problems in trigonometry

1. The answers to these calculations can be found on the cards below.

\[
\begin{align*}
6^2 - 3 \times 4 & \quad 10 + 8^2 \div 16 & \quad (9 - 12)^2 + 18 \div 3 \\
5 - (17 - 5^2) & \quad 28 + 2 \times -4 & \quad (4 + 3)^2 + 54 + -2
\end{align*}
\]

Work out the answers to the calculations. Now write down the letters from the cards with the answers on, starting with the smallest answer. What word have you written?

2. A ladder leans against a wall at an angle of 70°. The ladder is 4.5 m long. How high up the wall will the ladder reach?

3. The same ladder as in Q2 is now placed 0.9 m away from the wall. What angle does the ladder make with the ground?

4. A boat in the sea is 200 m from the base of a cliff. The angle of elevation of the top of the cliff is 9.9°. Find the height of the cliff to the nearest metre.

5. Wheelchair ramps can have a maximum incline of 3.8°. A 20 cm step needs a wheelchair ramp, as shown in the diagram. Find the measurements \(x\) and \(y\) to the nearest mm.

6. This trapezium is made from three equilateral triangles.
   a. Calculate the perpendicular height of the trapezium.
   b. Calculate the area of one equilateral triangle.
   c. Calculate the area of the trapezium.
1. ‘Doubling a number always makes the number larger.’
   Is this statement true? Justify your answer.

2. Explain why these statements cannot be quite true.
   a. I have flipped this 10p coin five times and have got five heads. I’m bound to get a tail this time!
   b. There’s no point playing Trivial Pursuit with my brother — he always gets the easy questions!

3. a. In an art class, 28 of the class of 31 students finished their project. A student was chosen at random from the class. What is the probability that it was a student who hadn’t finished their project?
   b. Flora plays darts. The probability that she hits the bullseye with a dart is \( \frac{2}{7} \). What is the probability that Flora misses the bullseye with her next dart?

4. A hair care company wants to find out which of their two new shampoos they should produce. They ask people to try out one of the shampoos and say whether they would recommend it to a friend. Here are the results of the trials.

<table>
<thead>
<tr>
<th>Shampoo</th>
<th>Number of people who tried shampoo</th>
<th>Number of people who said they would recommend it to a friend</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>250</td>
<td>150</td>
</tr>
<tr>
<td>B</td>
<td>320</td>
<td>180</td>
</tr>
</tbody>
</table>

   a. Write down the probabilities of a recommendation for each shampoo. Give your answers in their simplest form.
   b. Compare the probabilities and say which shampoo the company should produce.

5. A survey about netball was carried out on 100 Year 7 girls. Of the girls questioned, \( \frac{3}{10} \) said they preferred to be centre, \( \frac{1}{3} \) said they preferred to be a defender and \( \frac{1}{4} \) said they preferred to be an attacker. The rest of the girls had never played netball, so had no preference. What is the probability of a Year 7 girl never having played netball?

6. Read the information in Q5 again. The school where the survey was carried out has 160 girls in Year 7. How many of these are likely to have never played netball?

7. Nadia recorded the colours of the sweets in a packet of fruit gums. There are 40 sweets in a packet. The tally chart shows her results.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Tally</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>H</td>
</tr>
<tr>
<td>black</td>
<td>HH</td>
</tr>
<tr>
<td>orange</td>
<td>HHH</td>
</tr>
<tr>
<td>green</td>
<td>H</td>
</tr>
</tbody>
</table>

   a. Calculate the relative frequency of each colour.
   b. If Nadia bought five of these packets, how many sweets might she expect to be green?
15.2 Growing trees

1. The sum of two consecutive even numbers is 934. Use an algebraic method to find the value of these integers.

2. A bag contains five red balls and five blue balls. A ball is taken from the bag at random and replaced. Another ball is then taken from the bag and replaced.
   a. Draw a tree diagram to represent the possible outcomes and probabilities in this experiment.
   b. What is the probability of picking two red balls?
   c. What is the probability that both balls are the same colour?

3. A different bag contains three red balls and seven blue balls. A ball is taken from the bag at random and replaced. Another ball is then taken from the bag and replaced.
   a. Draw a tree diagram to represent the possible outcomes and probabilities in this experiment.
   b. What is the probability of picking two red balls?
   c. What is the probability that both balls are the same colour?

4. A bag contains five yellow balls and five black balls. A ball is taken from the bag at random and replaced. A second ball is then taken from the bag and replaced, and finally a third ball is taken from the bag and replaced.
   a. Copy and complete this tree diagram to represent the possible outcomes and probabilities in this experiment.
   b. What is the probability of picking three yellow balls?
   c. What is the probability that all three balls are the same colour?
   d. Jan says, 'The probability of getting two yellow and one black ball is $\frac{1}{8}$. Explain why Jan is wrong.'

5. A bag contains five red balls and five blue balls. A ball is taken from the bag at random but not replaced. Another ball is then taken from the bag and not replaced.
   a. Draw a tree diagram to represent the possible outcomes and probabilities in this experiment.
   b. What is the probability of picking two red balls?
   c. What is the probability that both balls are the same colour?
   d. Compare these probabilities with those in Q2, where the same experiment was carried out, but the balls were replaced each time.

6. A box contains 20 chocolates that all look the same. Six of the chocolates have a nut centre, six have a caramel centre and the rest have a fruit centre.
   Sheena takes three chocolates from the box.
   a. Draw a tree diagram to represent the possible outcomes and probabilities.
   b. What is the probability that all three chocolates have the same centre?
1. Write true or false for each of these.
   a. The mid-point of the line joining (6, 8) to (16, 8) is (8, 11).
   b. The mid-point of the line joining (−5, −6) to (−1, 6) is (−3, 0).

2. In a game of ‘double darts’, two darts are thrown at this dartboard. If both the darts stick in the board then the player adds the two numbers to get a total score. Before they throw their darts, a player chooses one of these rules for how to win.

   Rule 1: You win if the total score is 2.
   Rule 2: You win if the total score is more than 7.
   Rule 3: You win if the total score is less than 7.
   Rule 4: You win if the total score is even.

   a. Which rule will give you the most chance of winning?
   b. Which rule will give you the least chance of winning?
   c. Which rule is the fairest? What is the probability of winning with this rule?

3. A survey was carried out on 100 people. They were asked which toothpaste they preferred out of the four options shown. The tally chart shows the results.
   a. A chain of supermarkets sells 800,000 tubes of toothpaste each week.
      How many tubes of the most popular toothpaste should they expect to sell?
   b. A larger survey of 500 people was carried out.
      How many of these people would you expect to choose Lightening?

4. A car supermarket carried out two surveys with a representative sample of 100 people. Here are the results of Survey A, which asked about the make of people’s cars.

<table>
<thead>
<tr>
<th>Make</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford</td>
<td>32</td>
</tr>
<tr>
<td>Seat</td>
<td>12</td>
</tr>
<tr>
<td>Audi</td>
<td>8</td>
</tr>
<tr>
<td>Vauxhall</td>
<td>28</td>
</tr>
<tr>
<td>Other</td>
<td>20</td>
</tr>
</tbody>
</table>

Here are the results of Survey B, which asked about people’s favourite colour of car.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>17</td>
</tr>
<tr>
<td>Red</td>
<td>16</td>
</tr>
<tr>
<td>Silver</td>
<td>45</td>
</tr>
<tr>
<td>Blue</td>
<td>20</td>
</tr>
<tr>
<td>Other</td>
<td>2</td>
</tr>
</tbody>
</table>

The survey showed that people’s choice of colour did not affect their choice of make.

   a. The car supermarket expects to sell 700 cars next year.
      How many Ford cars should they expect to sell?
   b. Last year the supermarket sold 140 Vauxhall cars.
      How many of them would you expect to be silver?
   c. What is the probability of someone buying a blue Audi?
1. Write the number 0.067923 correct to three decimal places.

2. Jackie has two packs of cards. The cards have pictures of shapes on them.
   Pack A has equal numbers of cards with squares, triangles and circles on them.
   Pack B has equal numbers of cards with squares, triangles, circles and rectangles on them.
   Jackie says, ‘There is more chance of picking a square from pack A than pack B.’
   Is she right? Explain your answer.

3. Throughout the year a bingo hall records information about its customers.
   This is the summary for 2008.
   - 72% of customers are retired women. Of these, 25% eat lunch here and 75% don’t.
   - 6% of customers are retired men. Of these, 80% eat lunch here and 20% don’t.

   One Monday, the bingo hall has 400 customers.
   a. How many of them are likely to be retired women that have lunch at the hall?
   b. How many of them are likely to be retired men that have lunch at the hall?

4. Nick and Angus play a game with this fair, four-sided spinner.
   They spin the spinner two times each and they see what letters it lands on.
   Which of these rules makes the game fair? Explain your answers.
   - Rule 1: If it lands on a vowel, Nick gets two points.
     If it lands on a consonant, Angus gets two points.
   - Rule 2: If it lands on a vowel, Nick gets one point.
     If it lands on a consonant, Angus gets three points.
   - Rule 3: If it lands on a vowel, Nick gets three points.
     If it lands on a consonant, Angus gets one point.

5. Pavel is taking a driving theory test and a practical test. The tests are independent.
   The probability of passing the theory test is 0.95, and the practical test is 0.7.
   a. What is the probability that Pavel passes both tests?
   b. The test centre tests 6000 people. How many of them are likely to pass both tests?

6. Two boxes of chocolates contain chocolates that all look the same.
   Box A has 36 chocolates; 27 are soft centres and 9 are hard centres.
   Box B has 25 chocolates; 19 are soft centres and 6 are hard centres.
   a. Which box should you pick from to have the better chance of getting a hard centre?
   b. One of the hard centre chocolates from box A is eaten. Does this change your
      choice of box to have the better chance of getting a hard centre? Explain your answer.

7. Rico has a box of 10 unlabelled music CDs. Four of them are pop, three are rock and the rest
   are blues. He selects three CDs, one at a time, and puts them into his three-disc CD player.
   a. Draw a tree diagram to represent this information.
   b. Use your tree diagram to answer these questions.
      What is the probability that Rico has selected:
      i. three pop CDs
      ii. three rock CDs
      iii. one of each type of music?
15.5 Back to the Future

1. Use your calculator to work out $32 \times 2$ hr 26 mins.
   Give your answer in hours and minutes.

2. In a plant nursery trial, plant food A was compared with plant food B.
   Lily growers were given either plant food A or plant food B at random.
   
   - 30 out of the 70 nurseries given plant food A reported healthier lilies.
   - 30 out of the 80 nurseries given plant food B reported healthier lilies.

   Which of the following statements are true?
   A is the best plant food because $\frac{30}{70}$ is greater than $\frac{30}{80}$.
   A is the best plant food because $\frac{30}{80}$ is greater than $\frac{30}{70}$.
   A and B are equally good because 30 nurseries from each trial reported healthier lilies.

3. One box of chocolates (A) contains equal numbers of caramel centres, nut centres, truffle centres and fudge centres. Another box (B) contains equal numbers of caramel centres, nut centres and fudge centres.

   Bronwen says: 'There is more chance of picking a fudge centred chocolate from box A than box B.'
   Is she right? Explain your answer.

4. Imran recorded the colours of 40 cars as they passed his school. The tally chart shows his results.
   a. Calculate the relative frequency of 'other' coloured cars.
   b. If Imran recorded the colours of 400 cars, how many silver cars should he expect to see?

   Q5 to Q7 refer to this pack of cards.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Tally</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue</td>
<td>II</td>
</tr>
<tr>
<td>red</td>
<td>III</td>
</tr>
<tr>
<td>black</td>
<td>III</td>
</tr>
<tr>
<td>silver</td>
<td>III</td>
</tr>
<tr>
<td>other</td>
<td>III</td>
</tr>
</tbody>
</table>

5. The pack of cards is shuffled. One card is taken at random from the pack, its shape is noted, and then it is replaced in the pack. Another card is taken at random and its shape is noted.
   a. Draw a tree diagram to represent the possible outcomes and probabilities.
   b. What is the probability of picking two squares?

6. Raul has written a computer program that models the experiment in Q5. He runs the program 5000 times and gets two triangles 104 times. By using relative frequencies, say whether Raul's computer program is a fair one. Explain your answer.

7. The pack of cards is shuffled. One card is taken at random from the pack and its shape noted, but it is not replaced in the pack. Another card is taken at random and its shape is noted.
   a. Draw a tree diagram to represent the possible outcomes and probabilities.
   b. What is the probability of picking two squares?
You have twelve different designs of panes of glass.

When one of the panes is placed over another pane, a new pattern is formed. The table below shows 36 new patterns. Which six are wrong?
Copy these numbers. They are a limerick written in code!

Here are the clues to help you crack the code. The clues are all muddled up to make your life difficult!

**Beware!**
There is only one clue that you can start with, so look for that one very carefully.

<table>
<thead>
<tr>
<th>CLUES</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A + T = 15</td>
<td>C + A - B = 14</td>
<td>F = L + Y + 2N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W = 3T</td>
<td>$\sqrt{144} = T$</td>
<td>N + 2T = $x^2 - 869$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H = $\left(\frac{1}{4}\right)^{\frac{1}{2}}$</td>
<td>$Y = \frac{1}{12} \times T$</td>
<td>$G = \frac{1}{3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D^2 = 2T + 1</td>
<td>S = T + \sqrt{28}</td>
<td>$R^2 = 3(A + B + C + D) - D$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M = $\sqrt{T} + 4$</td>
<td>$X = I + T$</td>
<td>V = $\sqrt{AT} \times (E - R)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I = 1.5 \times T</td>
<td>$E = T + 2$</td>
<td>O = C + A + T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B = $\frac{T^2}{18}$</td>
<td>$U = A^3$</td>
<td>L = AT + 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Replace each number with the matching letter to reveal the limerick.
Unit 16 Homework investigation 3: What has 52 bones and 250 000 pores...?

The answers to these questions about the human body can be found on the yellow cards. Work out the answers to the questions and write down the letters from those cards. Now unscramble the letters to solve the puzzle.

The average adult has about 18 ft² of skin.
- a. Convert this to m². There are about 625 sweat glands per square inch of skin.
- b. How many sweat glands does the average adult have?

If a child of height 140 cm bangs a toe on a door, it takes \( \frac{1}{40} \) second for their brain to register the pain. At what speed does the pain signal travel from the toe to the brain? (Answer in km/h)

The average 12-year-old’s heart beats 80 times a minute. In every beat, the heart pumps 70 ml of blood out of the heart. How much blood is pumped out of the heart every day? (Answer in litres)

An adult’s lung, if stretched out, has the same area as a tennis court. A tennis court measures 24 m by 11 m. What is the area of an adult’s lung? (Answer in square feet to 3 s.f.)

15 million blood cells are produced and destroyed every second.
- a. How many are produced and destroyed every day? (Answer in standard form)
- b. One drop of blood contains 250 million blood cells. How many drops of blood are produced and destroyed every day?

A male passes about 13 000 gallons of urine in a lifetime. About how many men would it take to produce 2500 m³ of urine?

54 muscles are used to take a step forward. How many muscles are used to take 1000 steps forward?

Walking 34 miles burns up 1 pound (lb) of fat. How many km do you have to walk to burn up 1 kg of fat? (Answer to 3 s.f.)
Unit 16 Homework investigation 4: Have you met your match?

In this challenge you have to move matches to make squares. You must move the number of matches indicated in the question (no more and no less) and every match must be used as part of a square.

For example:
Move 2 to make 1

Answer:
2 matches moved, makes 1 square

Now it's your turn!

1. Move 2 to make 2
2. Move 2 to make 2
3. Move 2 to make 3
4. Move 3 to make 4
5. Move 2 to make 5
6. Move 3 to make 3
7. Move 3 to make 5
8. Move 3 to make 4
9. Move 4 to make 3
10. Move 4 to make 5
11. Move 3 to make 4

Warning: Don't play with matches.
Unit 16 Homework investigation 5: Simple symbol sums!

Thousands of years ago there were no numbers. Instead, people used fingers, sticks or rocks to represent numbers. This developed into a system of symbols, and finally into the numbers that we use today.

The Mayans devised a counting system using only three symbols. It dates back to the 4th century and was about 1000 years more advanced than the European systems of that time. This table shows the system they used, from 0 to 29.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
</tr>
</tbody>
</table>

This is how the Mayans would have written the calculations $6 + 8 = 14$

Use Mayan numbers to work out these.

1. 

2. 

3. 

The ancient Romans devised the Roman numeral system that you often still see today. This table shows the system they used.

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is how the Romans would have written the calculation $6 + 8 = 14$

$VI + VIII = XIV$

Use Roman numbers to work out these.

4. $XXII + XI$

5. $XL + XXVIII$

6. $CXXV + CCCLI$

7. $M + DX + XC + VI$

8. $CLIX - LXXXIII$

The modern binary number system was fully documented in the 17th century. It is a system that is used in computing and consists of using only 0s and 1s. This table shows the place value column headings for the binary system, and an example of how it is used.

<table>
<thead>
<tr>
<th>128</th>
<th>64</th>
<th>32</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use binary numbers to work out these.

10 $10010 + 100110$

11 $1101 + 111010$

12 $110 + 10111$

13 $1010 + 11001$
These puzzles involve moving silver (S) and copper (C) coins. The aim of the puzzles is to work out the smallest number of moves that are needed to reverse the positions of the silver and copper coins. You may find it useful to actually use 10p and 2p coins, and move them around to help you to work out how to solve the puzzles.

Jumping coins
For the first set of puzzles you must follow these rules:
- You can only move one coin at a time.
- You can move a coin into an adjacent space.
- You can jump a coin over one of the other type if there is a space immediately beyond it.
- You can't jump two places and you can't jump over a coin of the same type.

For example, it would take three moves to reverse these coins.

![Diagram of jumping coins example](image)

Work out the smallest number of moves needed to reverse each of these sets of coins.

1

2

3

4

5 There is a link between the number of silver and copper coins and the number of moves needed to reverse the coins. Can you work out the link?

Sliding coins
For the second set of puzzles you must follow these rules:
- You can only slide one coin at a time.
- You can slide a coin into an empty space.
- You can't jump a coin over any other coin.

For example, it would take three moves to reverse these coins.

![Diagram of sliding coins example](image)

Work out the smallest number of moves needed to reverse each of these sets of coins.

6

7

8
Welcome to Level Up Maths!
High interest maths that makes progression easy.
Level Up Maths is an inspirational new course for
today’s classroom, with levelled and sub-levelled
support to make progression easy.

Features of the Levels 6-8 Homework Book

- One-page homework for every lesson in the Level 6-8
  Pupil Book.
- Each homework starts with number skills practice.
- Questions levelled and sub-levelled, at the same levels as in the lesson.

Clearly referenced to the Pupil
LiveText CD-ROM

On the Pupil LiveText CD-ROM:
• the Level 6-8 Pupil Book on screen,
  so pupils don’t have to carry a Textbook home.
• interactive explanations
• audio glossary
• competitive games

Written by a large team of experienced teachers, Level Up Maths supports the new Programme of Study for Key Stage 3 Maths, and the Key Stage 3 Renewed Framework for Maths.
Level Up Maths is uniquely structured by Level Band to provide the best personalised progression path for every pupil.

Access Books

Textbooks

Other components include: Pupil Books, Teacher Planning and Assessment Packs, LiveText Whiteboard CD-ROMs, Online Assessment Services and Revision Guides.

www.heinemann.co.uk
01865 888060

Heinemann